Coherent oscillations and two-qubit dynamics with a PT symmetric polariton mixture

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Outline

- *Introduction to polaritons and application*
- *The model*
- *PT – symmetry breaking and fixed points*
- *Dynamics*
- *Simulating two-qubit SWAP gate*
- *Conclusions*

Introduction – Photons and Excitons

- Polaritons are hybrid quasi-particles consisting of an **exciton** and a **photon** which are **strongly coupled**.

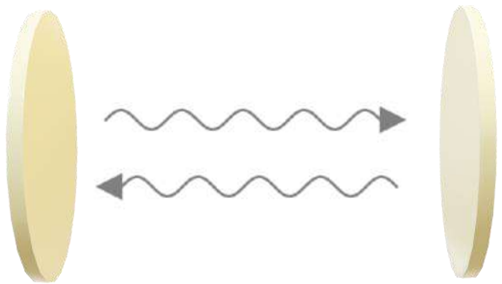
- When an e^- absorbs a photon it is possible that it "jumps" to a hole. Electrons jumping from hole to hole explain the current flow in semiconductors.

- An e^- which absorbs a photon may not jump into a hole. In this case the (excited) e^- and the hole become bound by their EM attraction. This **e^- - hole pair** is a quasi-particle called **exciton**.

Introduction - Microcavities

- ❑ Microcavity structures consist of two ***mirrors*** to form the cavity and a ***material*** to put in the cavity.

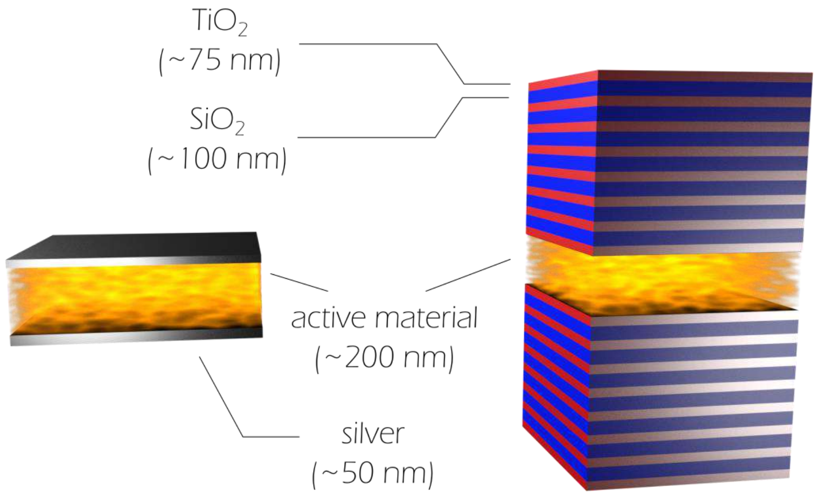
- ❑ Photons can be temporarily ***trapped*** in a microcavity.



- ❑ The quality of the reflectivity of a microcavity is determined by the $\boldsymbol{Q}$ **− factor**.

- ❑ Mirrors → Metal (silver, gold, aluminium)
  → **D**istributed **B**ragg **R**eflectors

- ❑ DBRs → High reflectivity up to 99.99%. Alternating High (TiO_2, Ta_2O_5) – Low (SiO_2) refractive index materials.



https://www.polaritonics.org/research/intro-to-polaritons

- ❑ Material → Organic (carbon based)
  → Inorganic (semiconductors, metals)

Introduction - Polaritons

- Polaritons are hybrid quasi-particles consisting of an **exciton** and a **photon** which are **strongly coupled**.

- Suppose a microcavity → Mirrors / Photon absorbing material

- A photon is absorbed, creating an exciton. Then it is emitted back to the cavity and the exciton decays. The photon, still bouncing in the cavity, can be again absorbed by the material creating another exciton.

- Fast energy exchange between the photon and the exciton → **strong coupling**

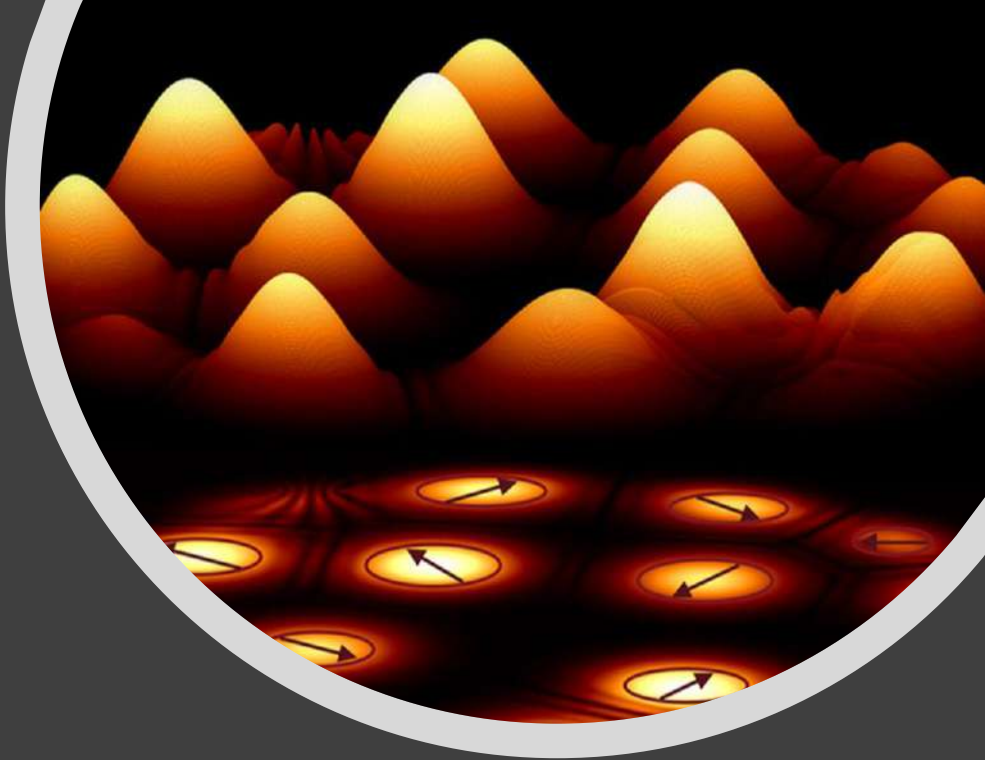
Polariton

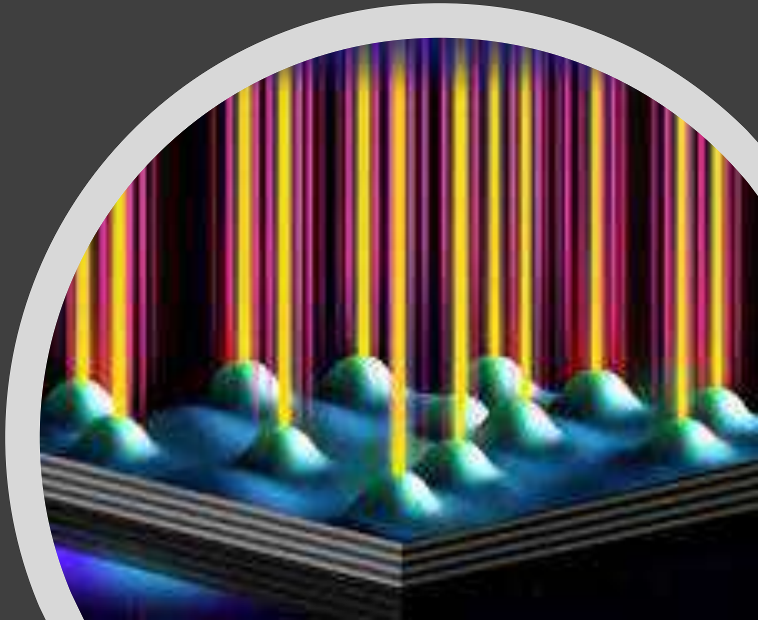
↓

Part-light and **part-matter** quasi-particle with properties of ***both***

Applications

- Polaritons simulators
- Polariton circuits
- Polariton lasers
- Polariton condensates





The polariton system

- Mixture of two polariton species in a double well potential.
- Each species (+) or (−) represents a polariton composed of a circularly polarized photon σ_+ or σ_-, coupled to cavity exciton.
- Mean field Gross-Pitaevskii ⟶ **DNLS**

$$i\hbar\dot{\psi}_L^{\pm} = \varepsilon_L^{\pm}\psi_L^{\pm} + g_s|\psi_L^{\pm}|^2\psi_L^{\pm} + g_c|\psi_L^{\mp}|^2\psi_L^{\pm} - J\psi_R^{\pm} + i\frac{\hbar}{2}[Q_L^{\pm}(n_L^{\pm}) - \kappa_L^{\pm}]\psi_L^{\pm}$$

$$i\hbar\dot{\psi}_R^{\pm} = \varepsilon_R^{\pm}\psi_R^{\pm} + g_s|\psi_R^{\pm}|^2\psi_R^{\pm} + g_c|\psi_R^{\mp}|^2\psi_R^{\pm} - J\psi_L^{\pm} + i\frac{\hbar}{2}[Q_R^{\pm}(n_R^{\pm}) - \kappa_R^{\pm}]\psi_R^{\pm}$$

$\psi_{L,R}^{\pm} = \sqrt{N_{L,R}^{\pm}}\,e^{i\phi_{L,R}^{\pm}}$ ⟶ complex WF

$N_{L,R}^{\pm} \equiv |\psi_{L,R}^{\pm}|^2$ ⟶ polariton populations

- The polariton BEC is dissipative with decay rate $\kappa_{R,L}^{\pm}$ (exciton recombination and cavity photon losses)
- The polariton population are continuously replenished via coupling to exciton reservoirs with rates $Q_R^{\pm}(n_R^{\pm})$

The exciton populations in each reservoir are:

$$\begin{aligned}\dot{n}_L^{\pm} &= P_L^{\pm} - \Gamma_L^{\pm}n_L^{\pm} - Q_L^{\pm}(n_L^{\pm})N_L^{\pm}\\ \dot{n}_R^{\pm} &= P_R^{\pm} - \Gamma_R^{\pm}n_R^{\pm} - Q_R^{\pm}(n_R^{\pm})N_R^{\pm}\end{aligned}$$

$\boldsymbol{P_{L(R)}^{+}}$: rate of exciton creation (laser pumping)

$\boldsymbol{\Gamma_{L(R)}^{+}}$: rate of decay of excitons in the reservoir

$\boldsymbol{Q_{L(R)}^{+}}$: rate of stimulated scattering of the reservoir excitons into the condensate of $N_{L(R)}^{+}$ polaritons.

The polariton system - Approximations

- The stimulated scattering rate can be approximated by a linear function of the of the reservoir exciton population:

$$Q^{\pm}_{L,R}(n^{\pm}_{L,R}) \simeq q^{\pm}_{L,R} n^{\pm}_{L,R}$$

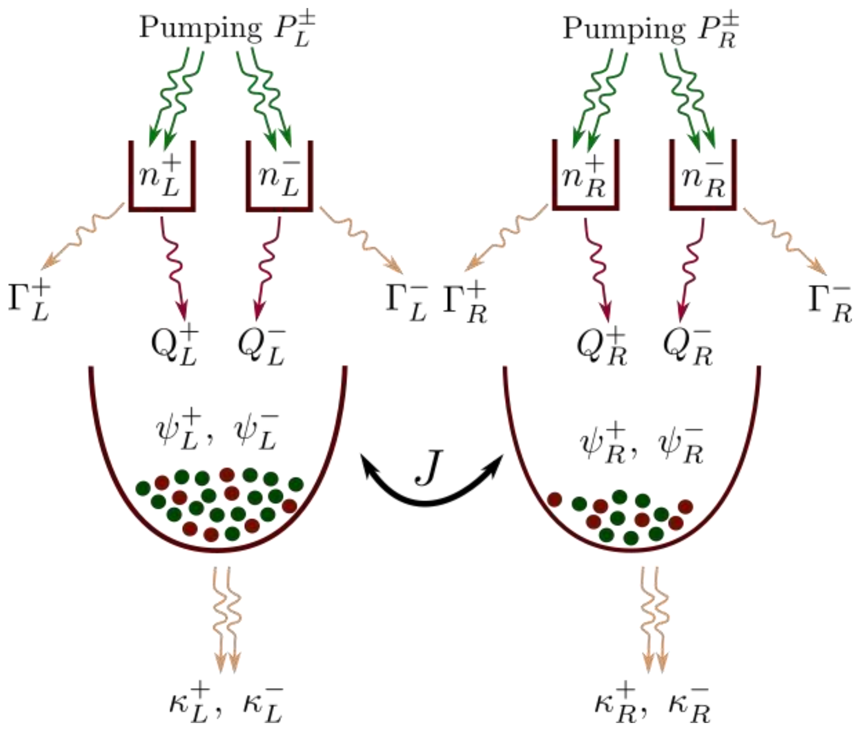
- Assume a sufficiently large reservoir → $\dot{n}^{\pm}_{L} \simeq 0$

$$n^{\pm}_{L,R} \simeq \frac{P^{\pm}_{L,R}}{\Gamma^{\pm}_{L,R} + q^{\pm}_{L,R} N^{\pm}_{L,R}}$$

- Assume that the reservoirs are weakly depleted by the coupling to the BEC:

$$q^{\pm}_{L,R} N^{\pm}_{L,R} \ll \Gamma^{\pm}_{L,R} \longrightarrow n^{\pm}_{L,R} = P^{\pm}_{L,R}/\Gamma^{\pm}_{L,R}$$

- The polariton gain/loss coefficients become: $2\gamma^{\pm}_{L,R} = [q^{\pm}_{L,R} P^{\pm}_{L,R}/\Gamma^{\pm}_{L,R} - \kappa^{\pm}_{L,R}]$

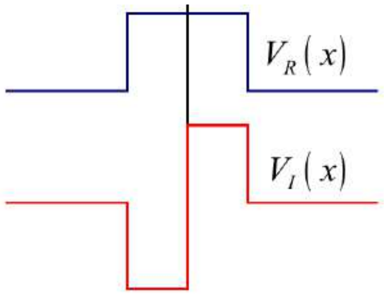


Rendering the system PT-symmetric

- PT symmetry conditions:

i. Reflection symmetric

ii. Maintain balanced gain and loss



- This is translated to the conditions:

$$\varepsilon_L^{\pm} = \varepsilon_R^{\pm} \quad ; \quad \gamma_L^{\pm} = -\gamma_R^{\pm}$$

or

$$\frac{q_{L,R}^{+} P_{L,R}^{+}}{\Gamma_{L,R}^{+}} - \kappa_{L,R}^{+} = -\left(\frac{q_{L,R}^{-} P_{L,R}^{-}}{\Gamma_{L,R}^{-}} - \kappa_{L,R}^{-}\right)$$

- Initially → equal # of particles in each well

$$N_L^- + N_L^+ = N_R^- + N_R^+ = N$$

- Switch on tunneling coupling *J*
- Increase the pumping rate near the left well to compensate losses
- Reduce the pumping rate near the right well in order to make losses dominant
- Engineer the structure in order to balance loss and gain

$$i\dot{\psi}_L^{\pm} = g_s|\psi_L^{\pm}|^2\psi_L^{\pm} + g_c|\psi_L^{\mp}|^2\psi_L^{\pm} - J\psi_R^{\pm} + i\gamma\psi_L^{\pm}$$

$$i\dot{\psi}_R^{\pm} = g_s|\psi_R^{\pm}|^2\psi_R^{\pm} + g_c|\psi_R^{\mp}|^2\psi_R^{\pm} - J\psi_L^{\pm} - i\gamma\psi_R^{\pm}$$

PT symmetry breaking and fixed points

- **Review of the linear system**: $g_s = g_c = 0$

- The $(+)$ and $(-)$ components decouple

- The PT symmetry breaking is determined by the eigenvalues of the corresponding Hamiltonian matrix:

$$\Lambda_{\pm} = \pm\sqrt{J^2 - \gamma^2}$$

- $J \leq \gamma$: PT symmetric phase Rabi-like oscillations → $\Omega = (\Lambda_+ - \Lambda_-)/2 = \sqrt{J^2 - \gamma^2}$

- $J > \gamma$: PT broken phase

- **Nonlinear system:** $g_s, \ g_c \neq 0$

- Symmetry breaking info will be obtained from the fixed points.

- We define new variables: $z^{\pm} = (N_L^{\pm} - N_R^{\pm})/N$

$$\Phi^{\pm} = \phi_R^{\pm} - \phi_L^{\pm}$$

- New equations:

$$\dot{z}^{\pm} = -2\sqrt{1-(z^{\pm})^2}\sin\Phi^{\pm} + 2\frac{\gamma}{J}$$

$$\dot{\Phi}^{\pm} = \frac{g_s}{J}z^{\pm} + \frac{g_c}{J}z^{\mp} + 2\frac{z^{\pm}}{\sqrt{1-(z^{\pm})^2}}\cos\Phi^{\pm}$$

- Dimensionless time: $t = J\tau$

PT symmetry breaking and fixed points

- **Fixed points:** $\dot{z}^{\pm} = \dot{\Phi}^{\pm} = 0.$

- Linearizing → stability eigenvalues λ_i

- Fixed point
  - stable $\mathrm{Re}[\lambda] < 0$
  - unstable $\mathrm{Re}[\lambda_i] > 0$
  - elliptic $\mathrm{Re}[\lambda] = 0$

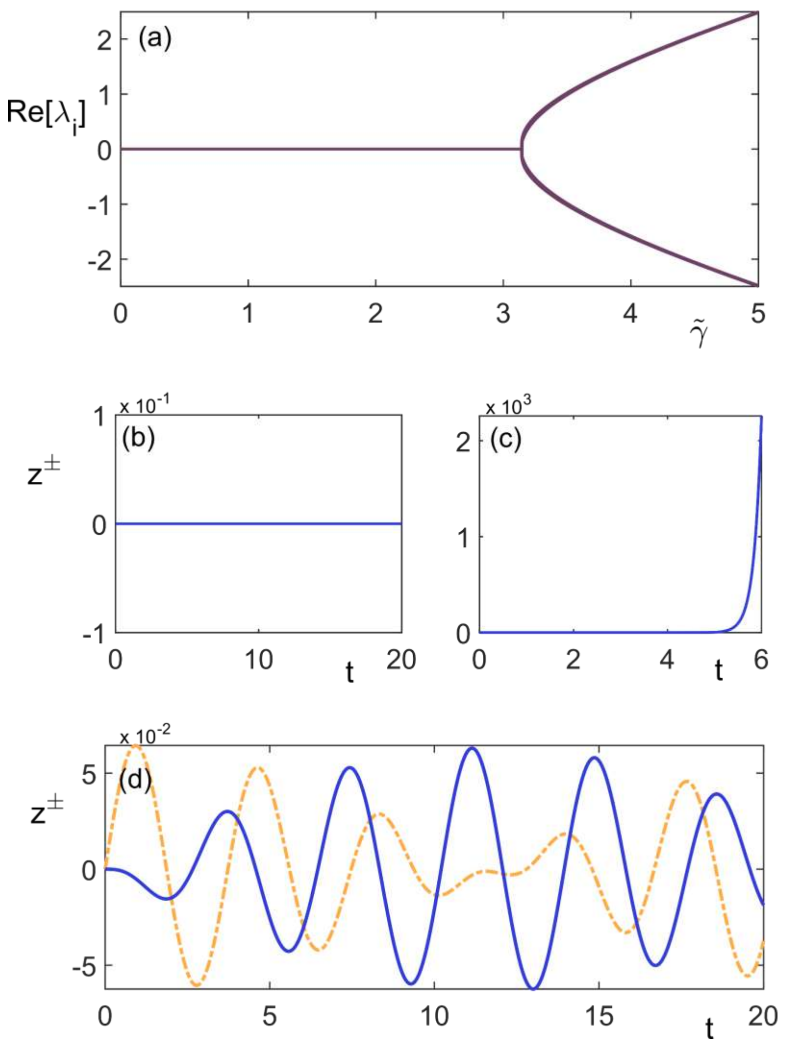
- A fixed point corresponds to the trivial solution

$$z_1^{\pm} = 0 \quad ; \quad \Phi_1^{\pm} = \pm \arcsin\left(\frac{\gamma}{J}\right)$$

with equal polariton population in each well.

- The stability eigenvalues are:

$$\lambda_i = \pm \frac{\sqrt{-4(J^2-\gamma^2)-2(g_s \pm g_c)\sqrt{(J^2-\gamma^2)}}}{J}$$

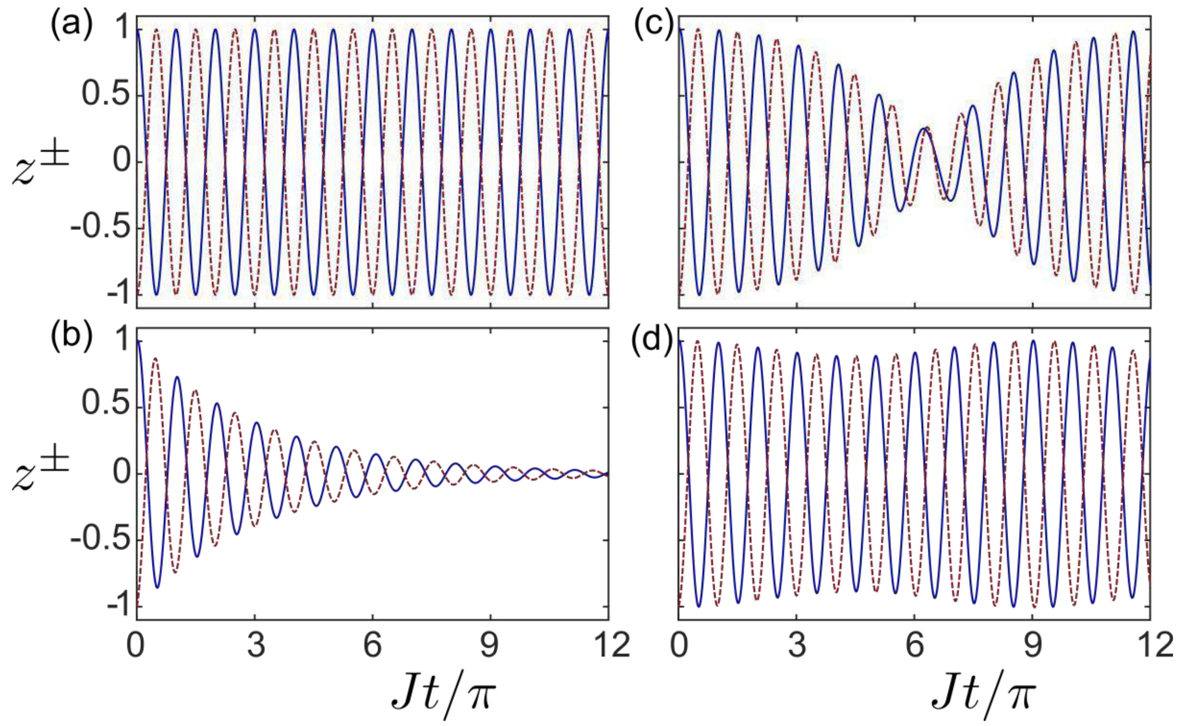


Dynamics

- **Initial configuration:** all (+) particles (L) / all (−) particles (R)

- Dynamics of the population imbalances

$$z^{\pm} = (N_L^{\pm} - N_R^{\pm})/N$$



a) Hermitian ($\gamma = 0$) → Rabi-like oscillations (non realistic for polariton systems)

b) The net loss is larger than the gain

$$|\gamma_R| = 0.2 > |\gamma_L| = 0.1$$

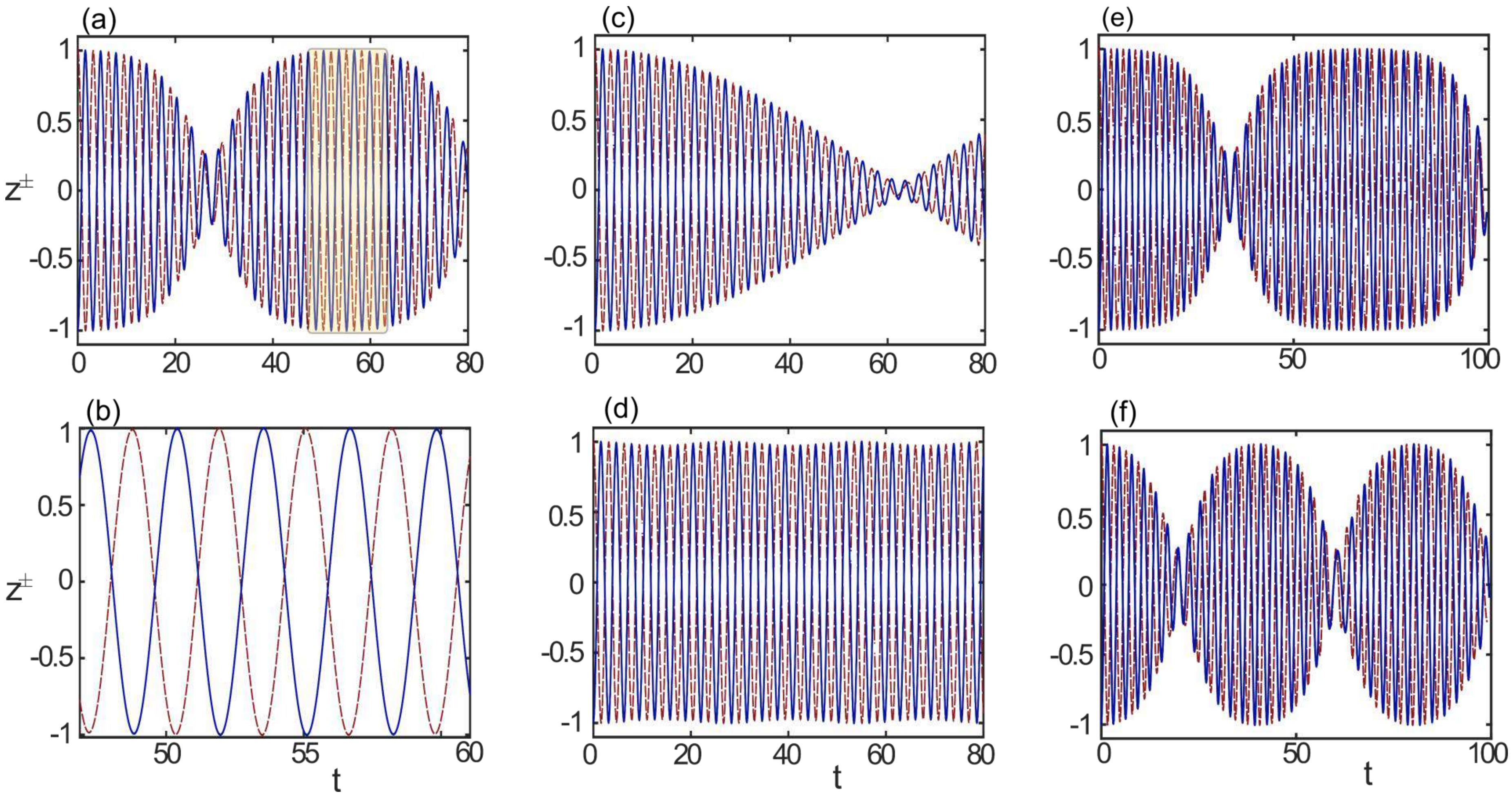
c) With suitable engineering $|\gamma_R| = |\gamma_L| = |\gamma|$

The PT symmetry preserves the coherent dynamics. Here, $g_s N = J,\ g_C N = 0.9J$

d) Same as (c) but for $g_s N = J,\ g_C N = 1.1J$

Dynamics

- Dependence of the dynamics on the nonlinearity strengths g_s, g_c.



Simulating a two-qubit system

- The two-species polariton system in a PT symmetric double well can be mapped onto a system of two qubits (or spin-1/2 particles) coupled via exchange interaction.

- This analogy links a classical system described by coupled-mode equations to a quantum system of coupled spins.

- The complete basis for a two-qubit system consists of four states

$$\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$$

- An arbitrary two-qubit state can be expanded as:

$$|\Psi\rangle = C_{\downarrow\downarrow}|\downarrow\downarrow\rangle + C_{\downarrow\uparrow}|\downarrow\uparrow\rangle + C_{\uparrow\downarrow}|\uparrow\downarrow\rangle + C_{\uparrow\uparrow}|\uparrow\uparrow\rangle$$

- We associate the $(+)$ component of the polaritons in each well with the amplitude of the $|\uparrow\rangle$ state of the spin [accordingly, for the $(-)$ component].

- The superposition amplitudes C given by:

$$C_{\downarrow\downarrow} = c_L^- c_R^-,\ C_{\downarrow\uparrow} = c_L^- c_R^+,\ C_{\uparrow\downarrow} = c_L^+ c_R^-,\ C_{\uparrow\uparrow} = c_L^+ c_R^+$$

with:

$$c_L^\pm \equiv \frac{\psi_L^\pm}{\sqrt{|\psi_L^+|^2 + |\psi_L^-|^2}},\quad c_R^\pm \equiv \frac{\psi_R^\pm}{\sqrt{|\psi_R^+|^2 + |\psi_R^-|^2}}$$

Our aim is to realize the SWAP gate between the two spins representing qubits

Simulating a two-qubit system - Fidelity

- Simplest scenario → $g_s = g_c = \gamma = 0$

Initial conditions

- $N_L^+ = N_R^+ = 0 \; ; \; N_L^- = N_R^- = \frac{N}{2}$
- $N_L^- = N_R^+ = \frac{N}{2} \; ; \; N_L^+ = N_R^- = 0$
- $N_L^- = N_R^+ = 0 \; ; \; N_L^+ = N_R^- = \frac{N}{2}$
- $N_L^+ = N_R^+ = \frac{N}{2} \; ; \; N_L^- = N_R^- = 0$

- These initial condition can be represented as input vectors

$$u_{\downarrow\downarrow} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix},\ u_{\downarrow\uparrow} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix},\ u_{\uparrow\downarrow} = \begin{bmatrix}0\\0\\1\\0\end{bmatrix},\ u_{\uparrow\uparrow} = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

- **Average Fidelity**

We characterize gate transformation using the fidelity formula

$$F = \frac{1}{n(n+1)}\left[\mathrm{Tr}(MM^\dagger) + |\mathrm{Tr}(M)|^2\right]$$

- n: Matrix dimensions (4 in our case)
- $M = U_0^\dagger U$
- $U_0^\dagger$: desired quantum gate
- U : actual transformation we get from simulations

For any initial state $|\Psi\rangle$, the desired final state is $U_0|\Psi\rangle$ while the actual state is $U|\Psi\rangle$, and F gives the fidelity of the transformation averaged over all the input states.

Desired quantum gate

❑ Simplest scenario → $g_s = g_c = \gamma = 0$

❑ The dynamics are analytically solvable and for each input state the time-dependent C amplitudes of the output state

$$|\Psi\rangle = C_{\downarrow\downarrow}|\downarrow\downarrow\rangle + C_{\downarrow\uparrow}|\downarrow\uparrow\rangle + C_{\uparrow\downarrow}|\uparrow\downarrow\rangle + C_{\uparrow\uparrow}|\uparrow\uparrow\rangle$$

are:

	$u_{\downarrow\downarrow}$	$u_{\downarrow\uparrow}$	$u_{\uparrow\downarrow}$	$u_{\uparrow\uparrow}$
$C_{\downarrow\downarrow}$	e^{2iJt}	$i\sin(Jt)\cos(Jt)$	$i\sin(Jt)\cos(Jt)$	0
$C_{\downarrow\uparrow}$	0	$\cos^2(Jt)$	$-\sin^2(Jt)$	0
$C_{\uparrow\downarrow}$	0	$-\sin^2(Jt)$	$\cos^2(Jt)$	0
$C_{\uparrow\uparrow}$	0	$i\sin(Jt)\cos(Jt)$	$i\sin(Jt)\cos(Jt)$	e^{2iJt}

❑ *The table represents the elements of the transformation matrix U.*

❑ We choose as interaction time half the period of the oscillation:

$$tJ = \pi/2$$

↓

$$U_{\text{SWAP}} = -\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

❑ **We will use this as the desired quantum gate**

State transfer with polaritons

- Linear PT system ⟶ $g_s = g_c = 0, \ \gamma \neq 0$
- Still this system is analytically solvable and we can find the U transformation matrix.
- Selecting as interaction time $tJ = \pi/2$ we find the fidelity of the state transfer for different values of loss/gain:

- $\gamma/J = 0.1 \to F = 0.992$
- $\gamma/J = 0.3 \to F = 0.935$

- Nonlinear PT system ⟶ $g_s = g_c = 1, \ \gamma \neq 0$
- No analytic solution ⟶ Numerical evaluation of Cs.
- Selecting as interaction time $tJ = \pi/2$ we find the fidelity of the state transfer for different values of loss/gain:

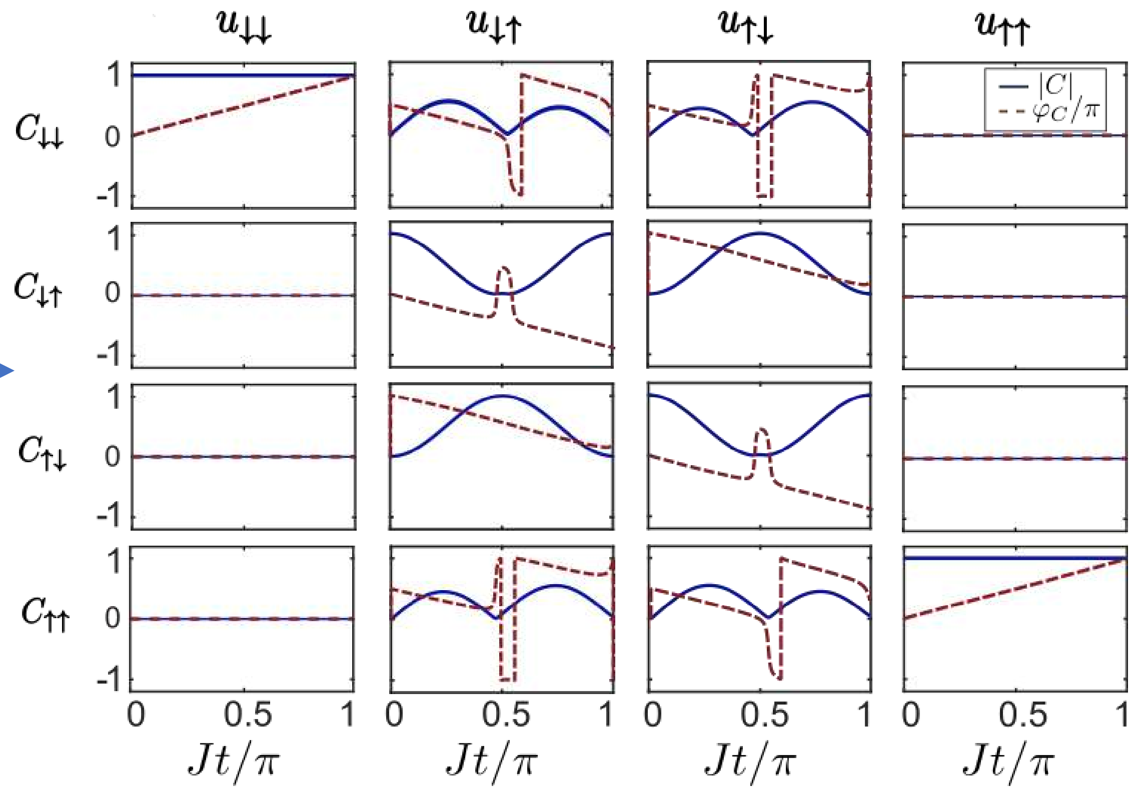
- $\gamma/J = 0.1 \to F = 0.991$
- $\gamma/J = 0.3 \to F = 0.922$
- $\gamma/J = 0.5 \to F = 0.799$

State transfer with polaritons

- Nonlinear PT system → $g_s \neq g_c,\ \gamma \neq 0$

- This is the most general case for polaritons

- Parameters: $g_s = 1,\ g_c = 0.5,\ \gamma = 0.1$ → $\mathbf{F = 0.982}$



- By increasing the difference between g_s and g_c

  F ↓ (but slowly)

- Example: $g_s = 1,\ g_c = 0.1,\ \gamma = 0.1 \rightarrow F = 0.963$

  $g_s = 1,\ g_c = 0.1,\ \gamma = 0.3 \rightarrow F = 0.867$

Conclusions

- We have considered a two-species polariton mixture with self- and cross-interaction nonlinearities in a PT – symmetric double well structure.

- We have shown the existence of long-term, coherent (Rabi-like) oscillations of the two polariton components in the presence of self- and cross-interactions, when the system lies in the PT-symmetric phase.

- The modulation of the cross-interaction strength, with respect to the cross-interactions strength induces different patterns in the dynamics of the polariton populations.

- We have shown that this system can be mapped onto a quantum system of two-qubits (or two spin-1/2 particles) via exchange interaction.

- We have found that this –essentially classical system of two coupled BECs- can simulate quantum state transfer with good fidelity.

Work under review: P.A.K, G. M. Nikolopoulos, and D. Petrosyan, *"Coherent population oscillations and an effective spin-exchange interaction in a PT symmetric polariton mixture"*, arXiv: 1908.00851v1.

Perspectives and work in progress

- Our work can contribute to the pursue of analog simulations of interacting few- and many-body quantum systems with coupled polaritons in polariton lattices.

- *Currently: Dynamics and state transfer in PT-symmetric three level systems in open and closed geometries and diamond lattices.*

- *Future: Consider the full system, add driving….*

In collaboration with:

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- **George Nikolopoulos**

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- **Manos Paspalakis**

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Thank you!