Acoustic subwavelength networks for waveguiding

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Waveguiding

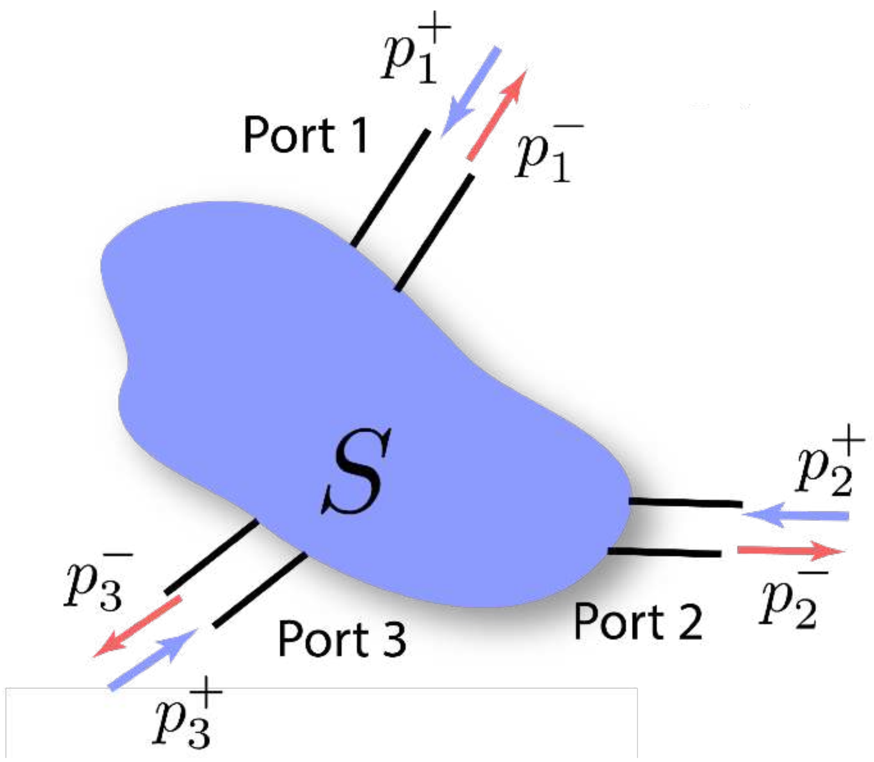
- Great challenge for many physical applications
- Numerous works on the design of multi-port devices
  - power divider and combiner
  - multiplexer systems

- Microwave : power divider and combiner, multiplexer, circulator, filter couplers …

- Realization by different ways :
  - resonant structures
  - photonic crystals
  - metamaterials,
  - multimode interferences,
  - nonuniform waveguide networks

3-port network

Scattering characterization



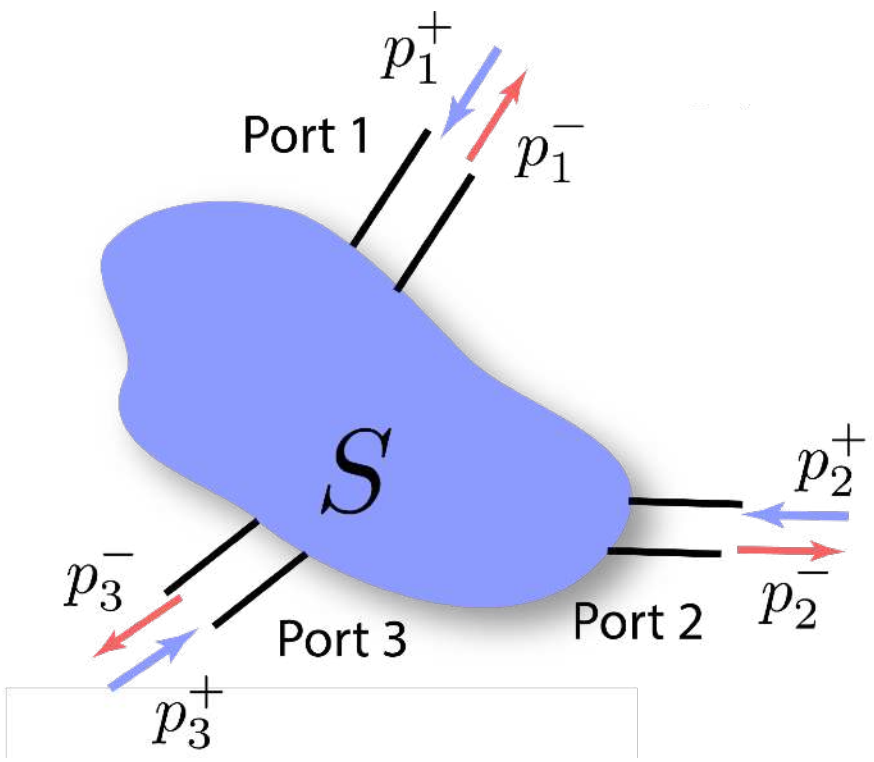
Scattering matrix

$$\begin{pmatrix} p_1^- \\ p_2^- \\ p_3^- \end{pmatrix} = \begin{pmatrix} r_1 & t_{12} & t_{13} \\ t_{21} & r_2 & t_{23} \\ t_{31} & t_{32} & r_3 \end{pmatrix} \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix} = S \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix},$$

↑ Output waves ↑ Input waves

3-port network

Scattering characterization



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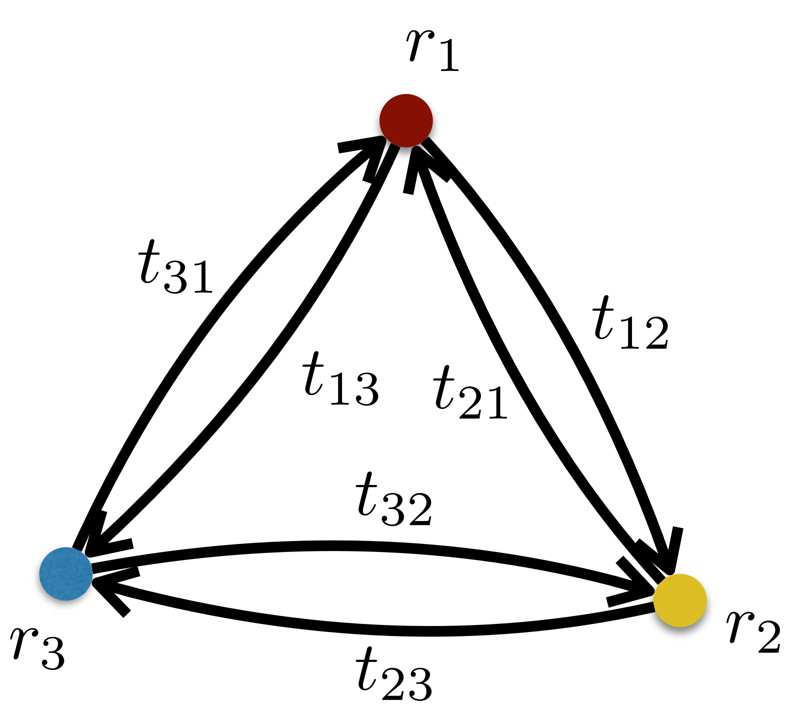
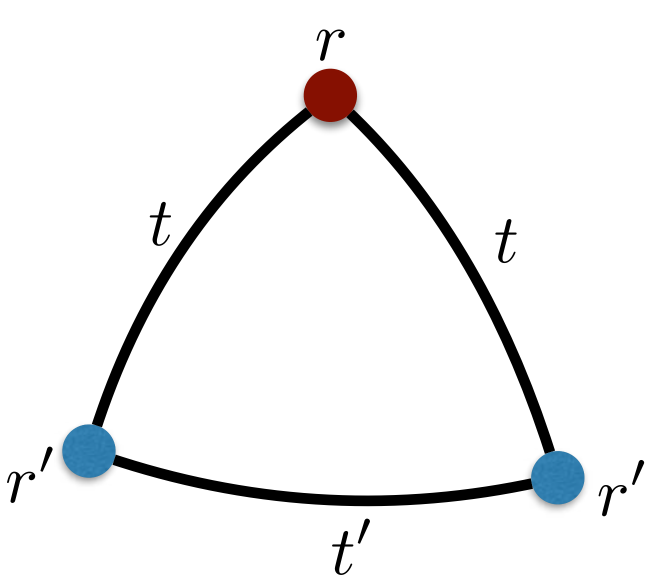
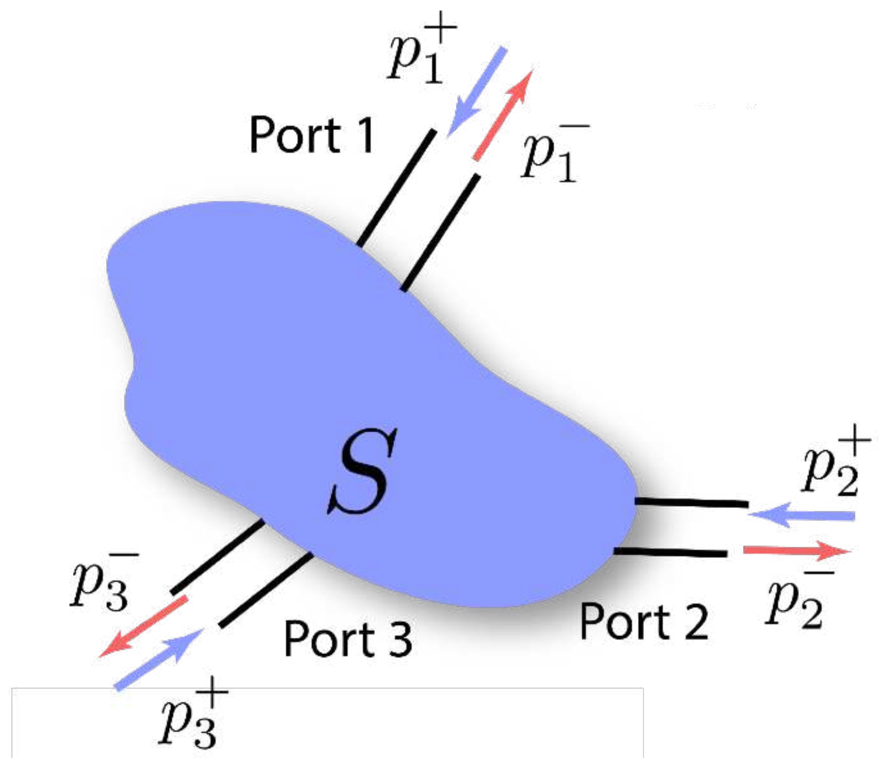




3-port network

Scattering characterization



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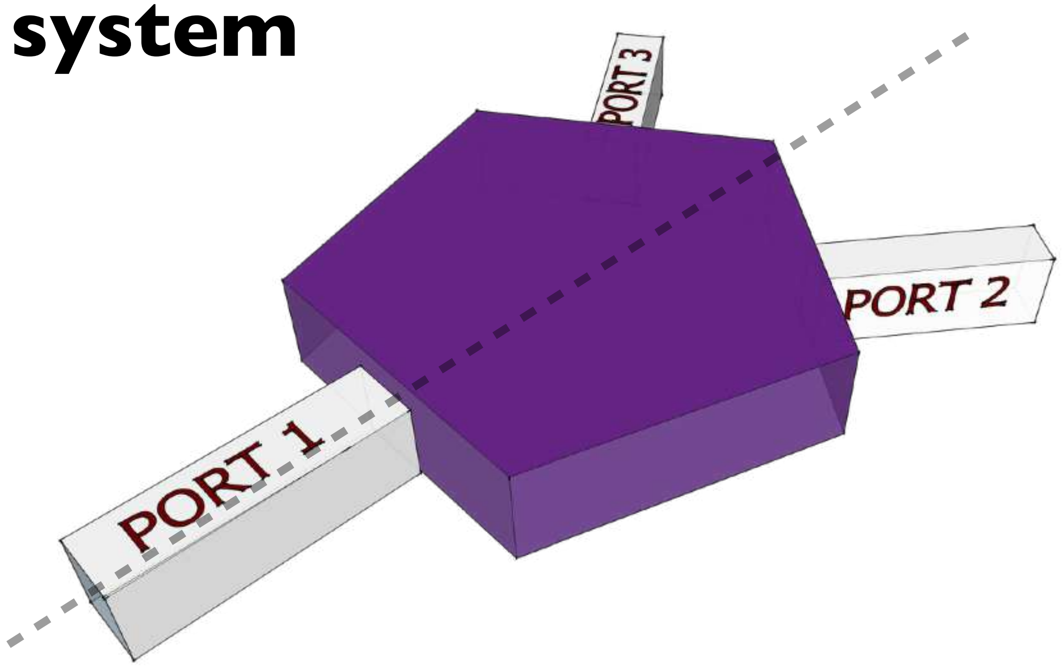
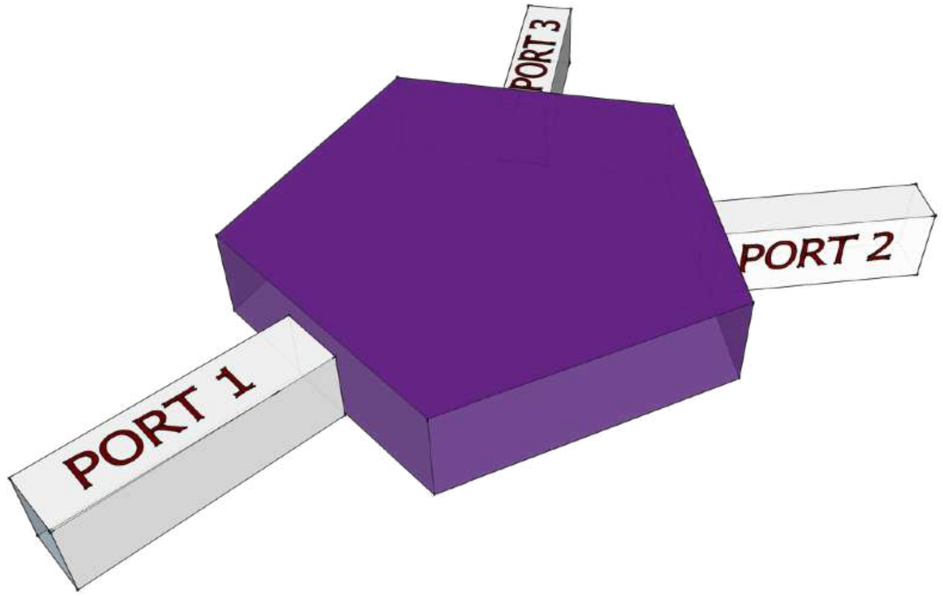
Output waves　　Input waves

Scattering characterization : Mirror symmetry and reciprocal system



$$\begin{pmatrix} p_1^- \\ p_2^- \\ p_3^- \end{pmatrix} = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix} \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix} = S \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix},$$

3-port network with mirror symmetry



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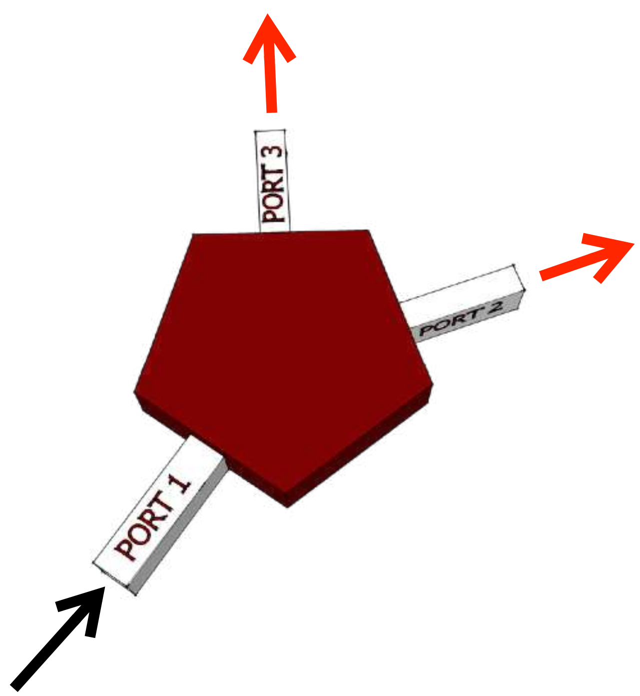
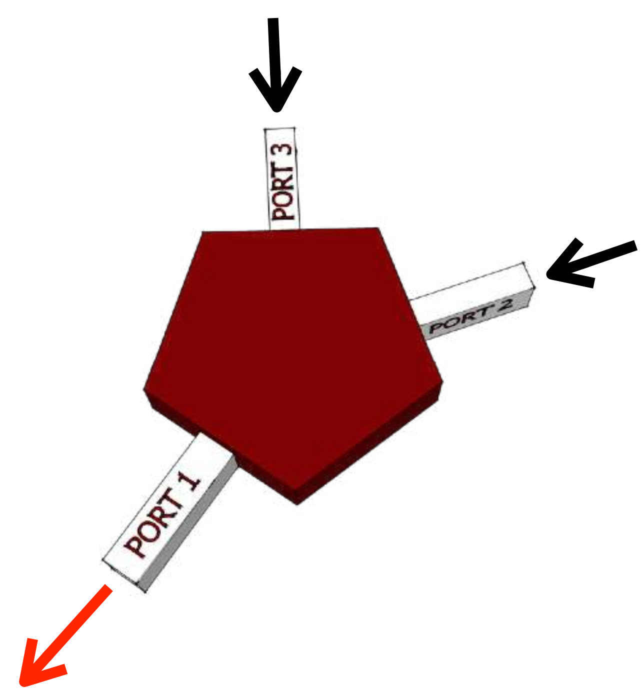
Splitter (divider)



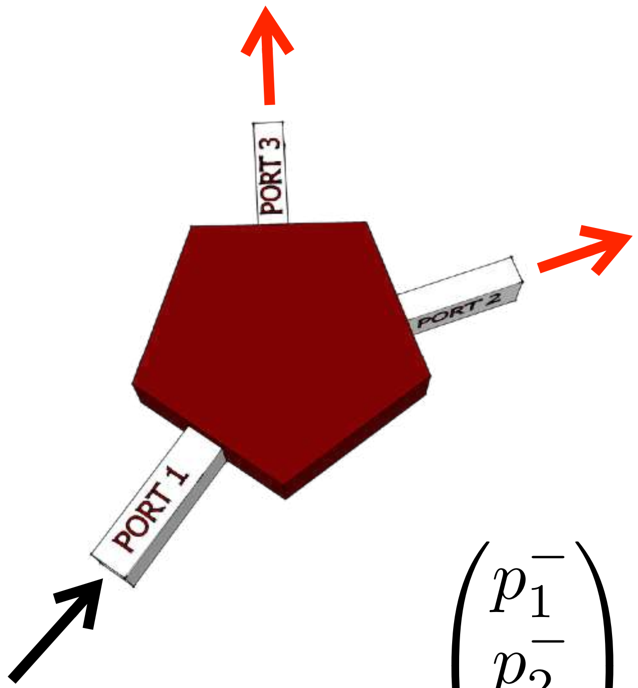
Combiner



➡ Use of the scattering matrix to find the conditions for the combiner or the splitter

3-port network with mirror symmetry

Splitter systems



$$\begin{pmatrix} p_1^- \\ p_2^- \\ p_3^- \end{pmatrix} = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix} \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix}$$

Conditions :

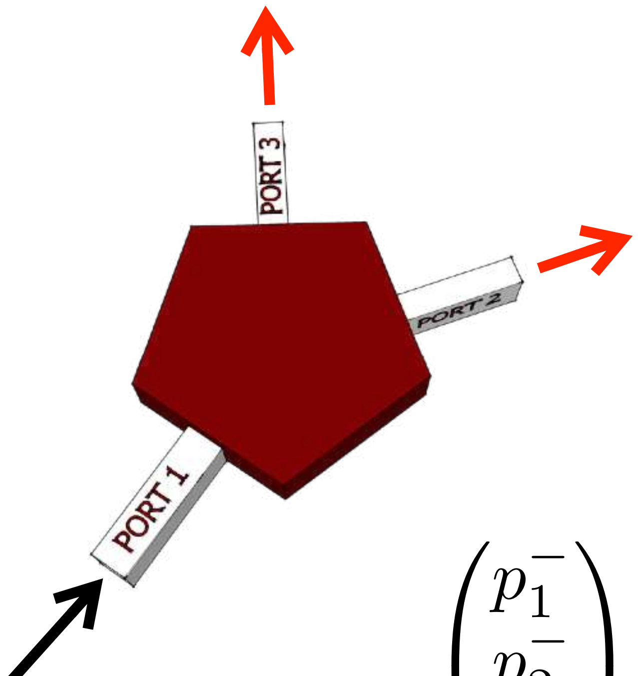
$p_2^+ = p_3^+ = 0$ and $p_1^- = 0$

↓

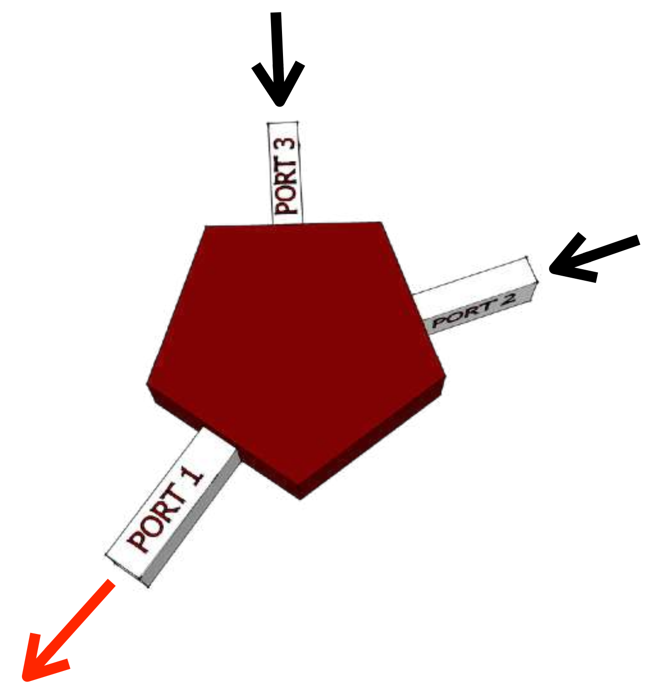
$$r = 0$$
$$\Theta = 2|t|^2$$

3-port network with mirror symmetry

Splitter systems



Combiner systems



$$\begin{pmatrix} p_1^- \\ p_2^- \\ p_3^- \end{pmatrix} = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix} \begin{pmatrix} p_1^+ \\ p_2^+ \\ p_3^+ \end{pmatrix}$$

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$r = 0$

$\Theta = 2|t|^2$

Conditions :

$p_1^+ = 0$ and $p_2^- = p_3^- = 0$

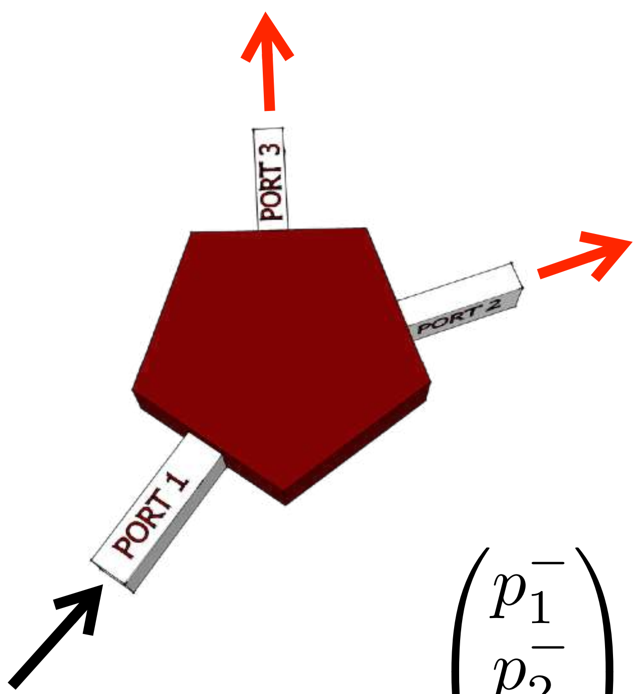
↓

$r' + t' = 0$

$\Theta = 2|t|^2$

3-port network with mirror symmetry

Splitter systems



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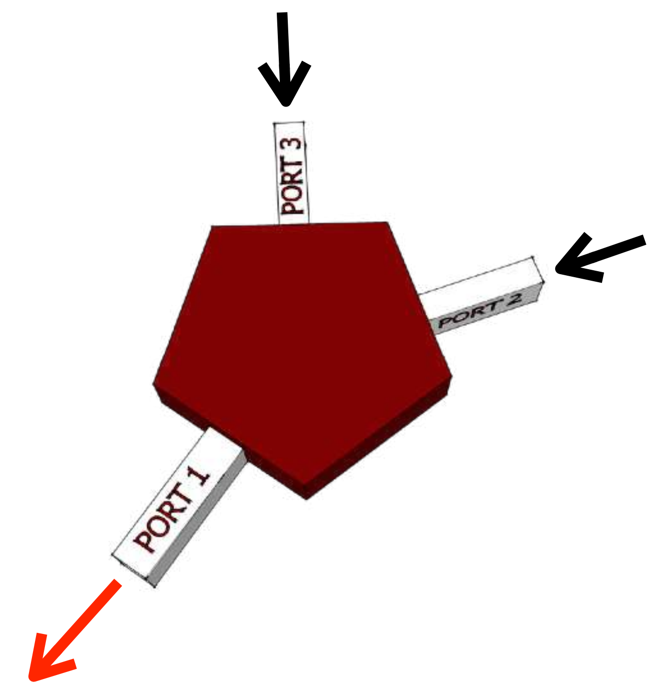

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Conditions :

$p_1^+ = 0$ and $p_2^- = p_3^- = 0$

↓

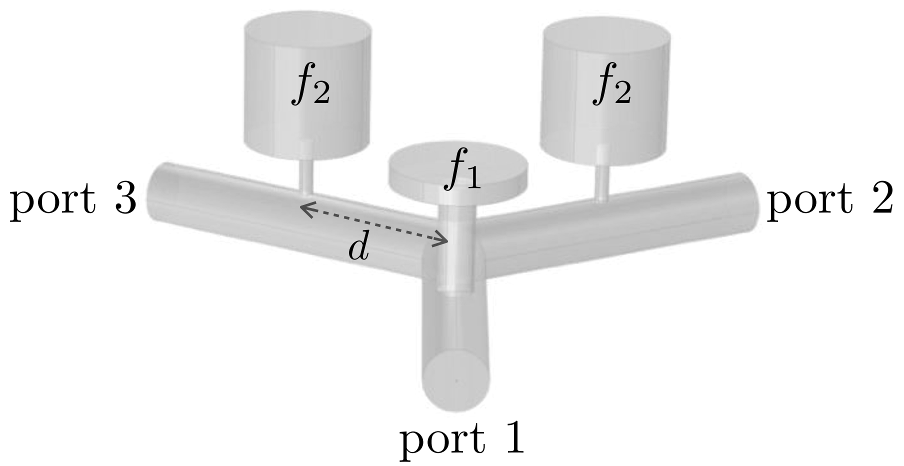
$r' + t' = 0$

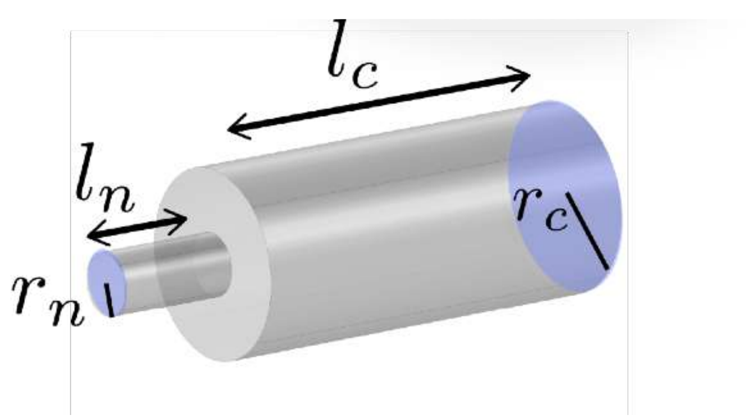
$\Theta = 2|t|^2$

Time reversal symmetry

↔

Acoustic subwavelength device





Tuning transmission and reflection by using
Induced Transparency Resonance
coming form the interaction of the resonances

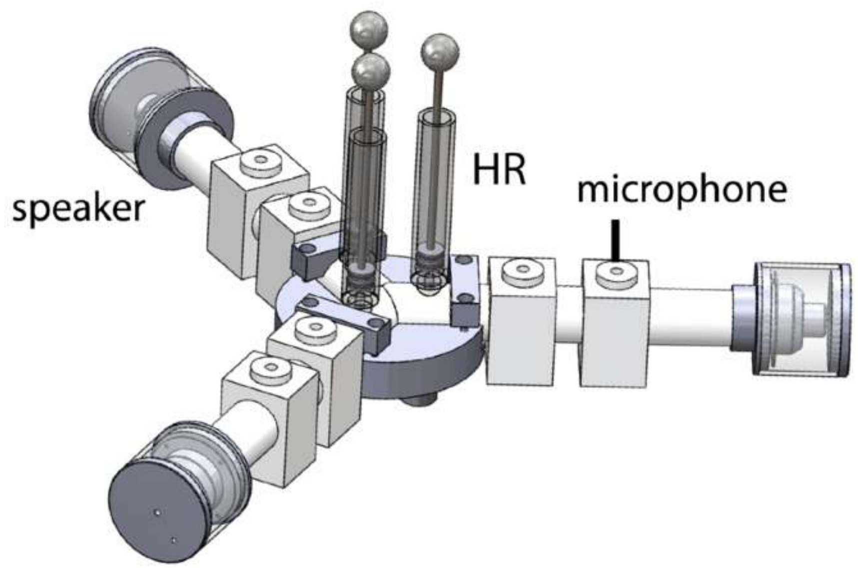
→ specific frequency f^*

r, t, r' and t' depend on

f	wave frequency
$f_1,\ f_2$	resonance frequencies
d	distance from the Y-connection
$R_1,\ R_2$	losses due to the resonators
$\gamma_1,\ \gamma_2$	coupling between waveguide and resonators

Analytical expressions
using transfer matrix
single mode propagation

Experimental results

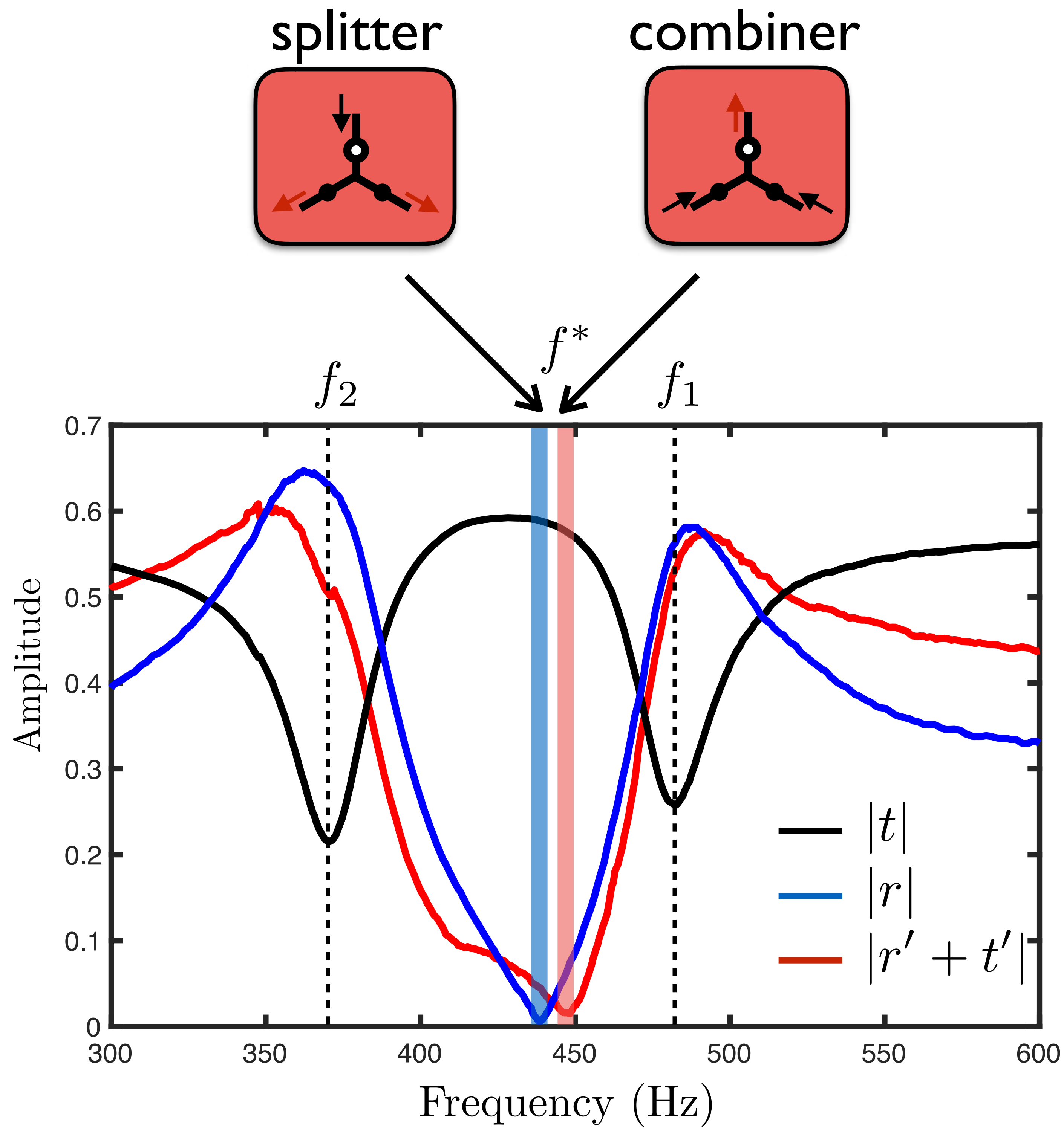


Geometry characteristics

$d = 5\,\mathrm{cm}$

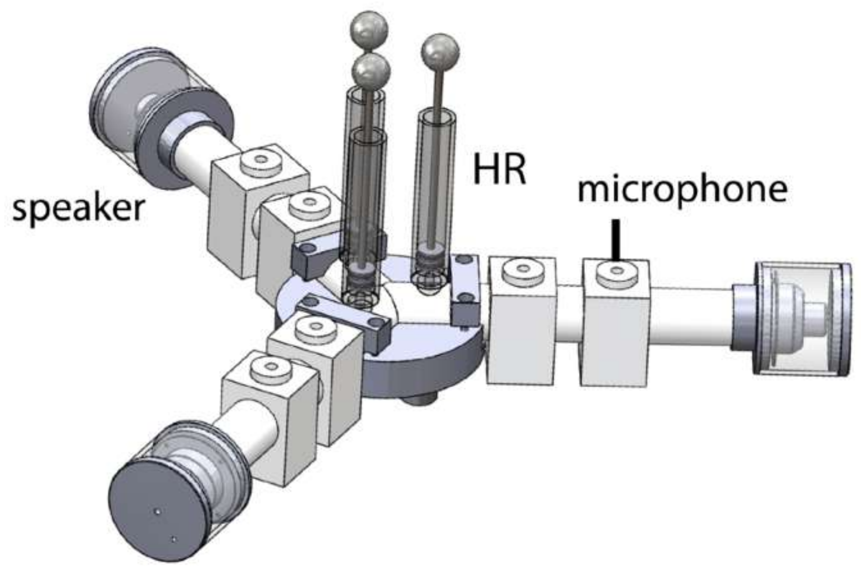
$\ell_{c1} = 2.9\,\mathrm{cm}$

$\ell_{c2} = \ell_{c3} = 4.9\,\mathrm{cm}$



Acoustic 3-port network

Experimental results



Geometry characteristics

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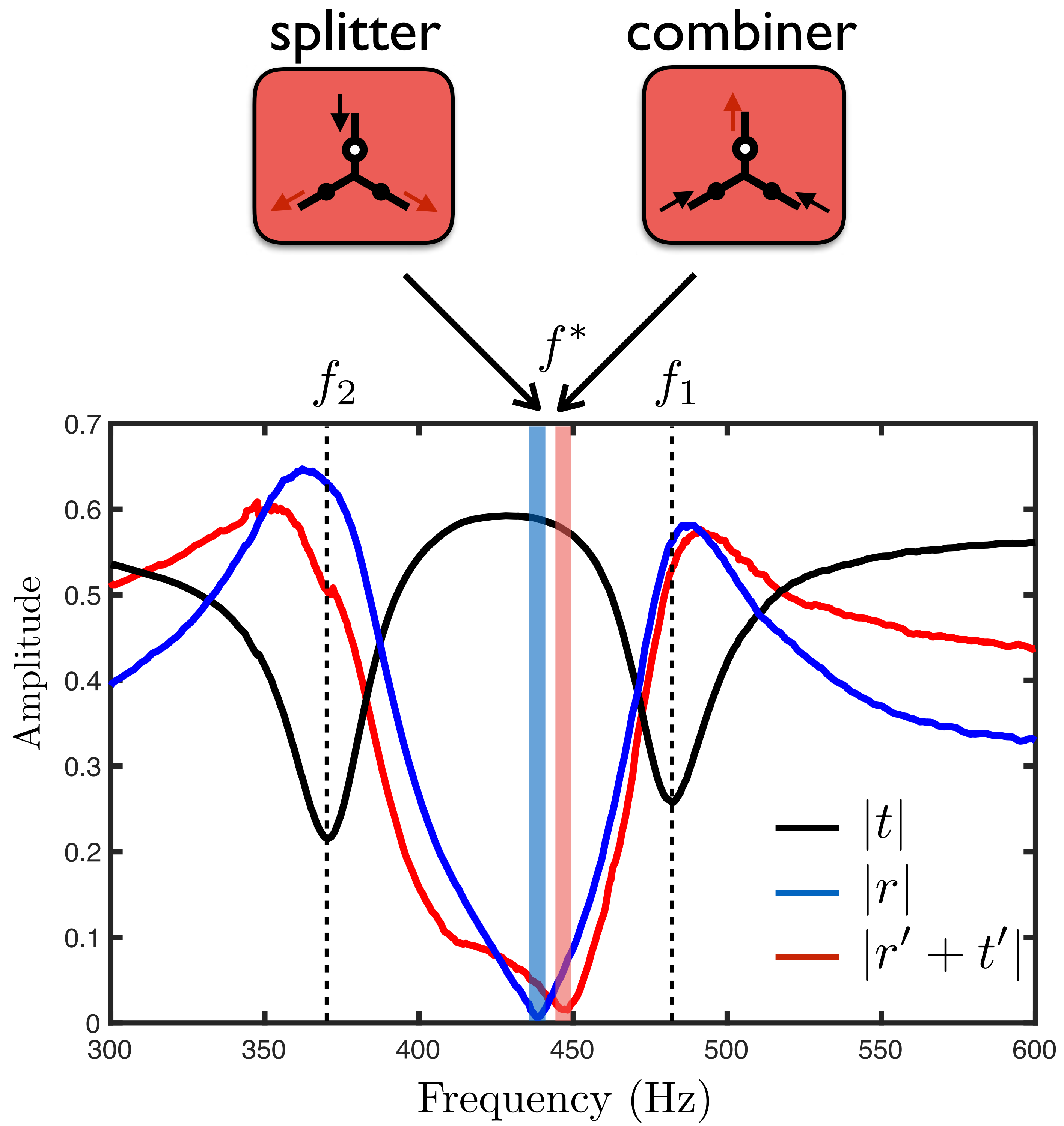
subwavelength system

$$\lambda/(2d) \approx 7.5$$

losses



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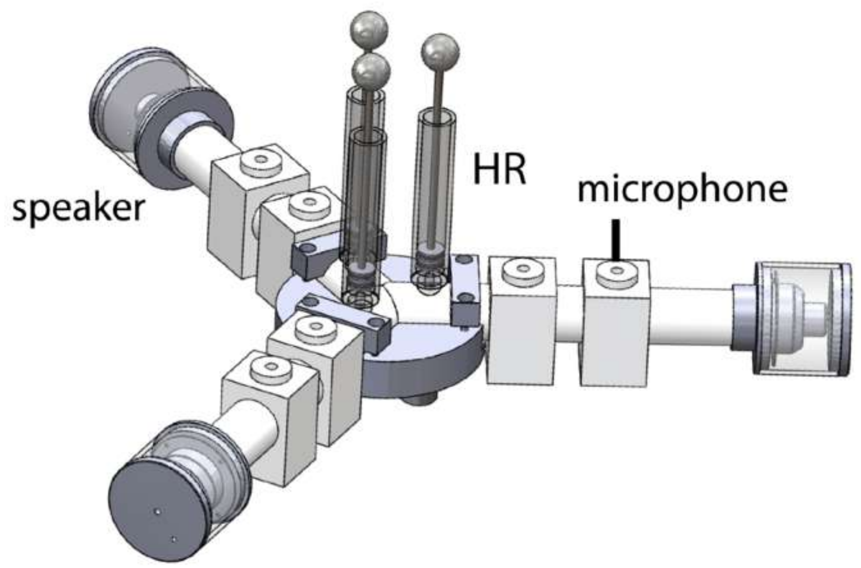
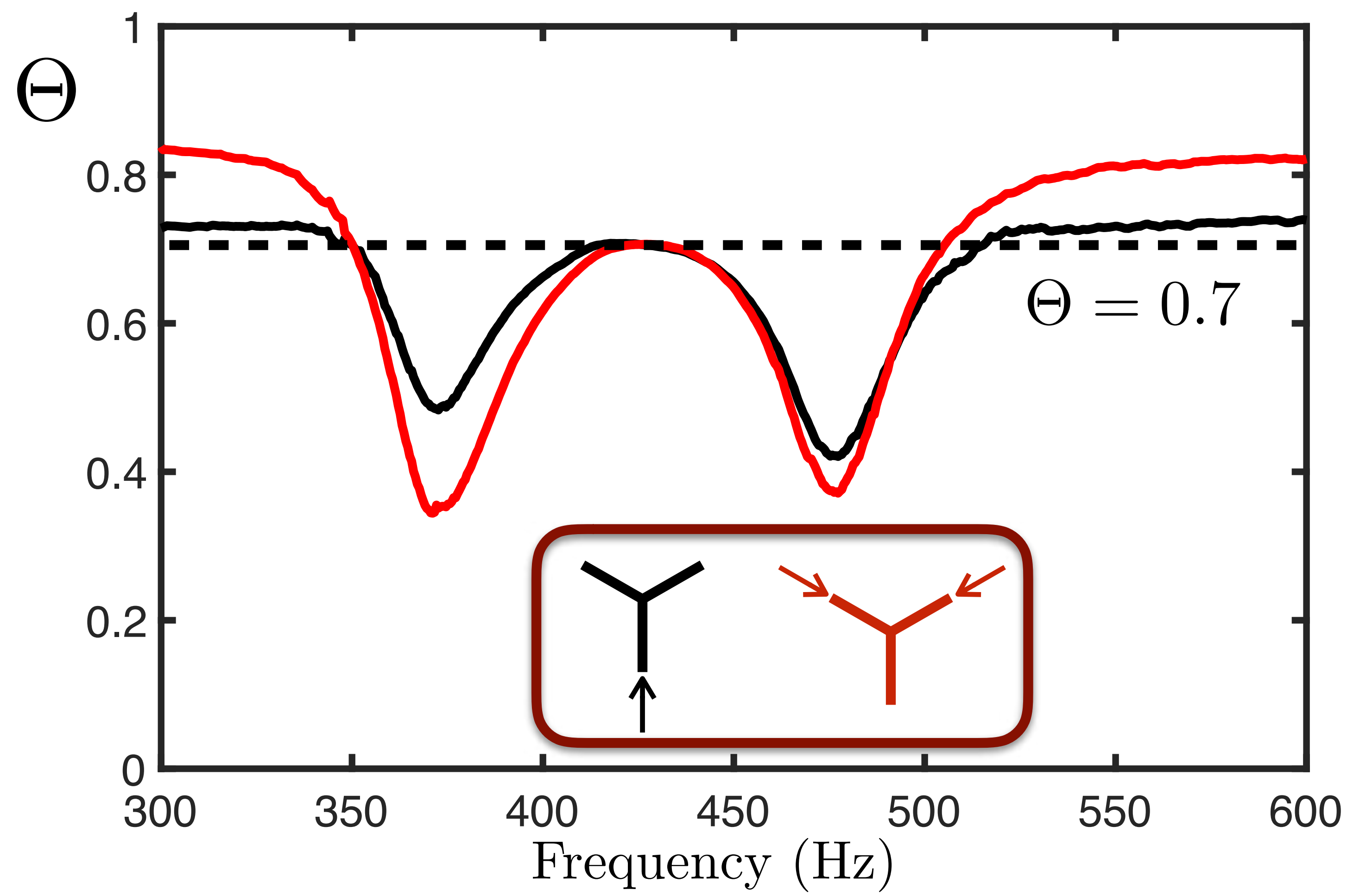


Output to input power ratio $\Theta = \frac{\sum_i |p_i^-|^2}{\sum_i |p_i^+|^2}$



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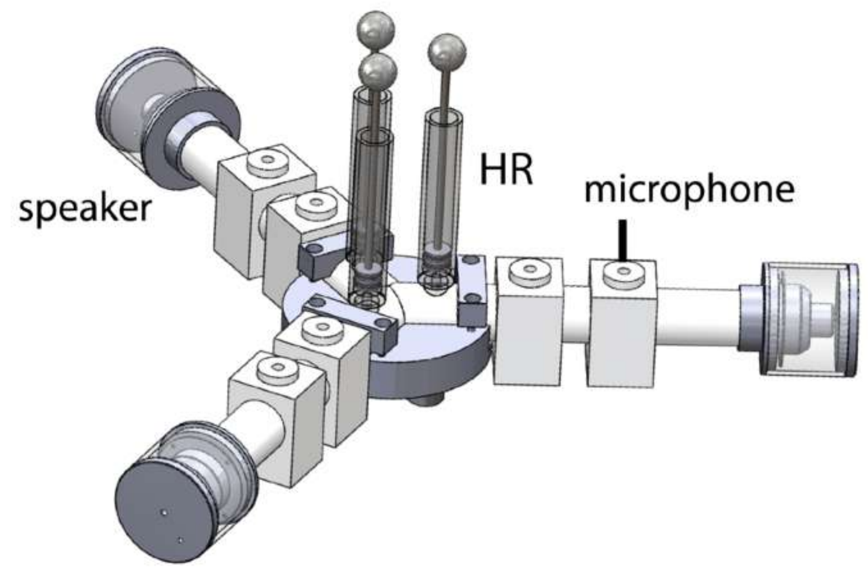
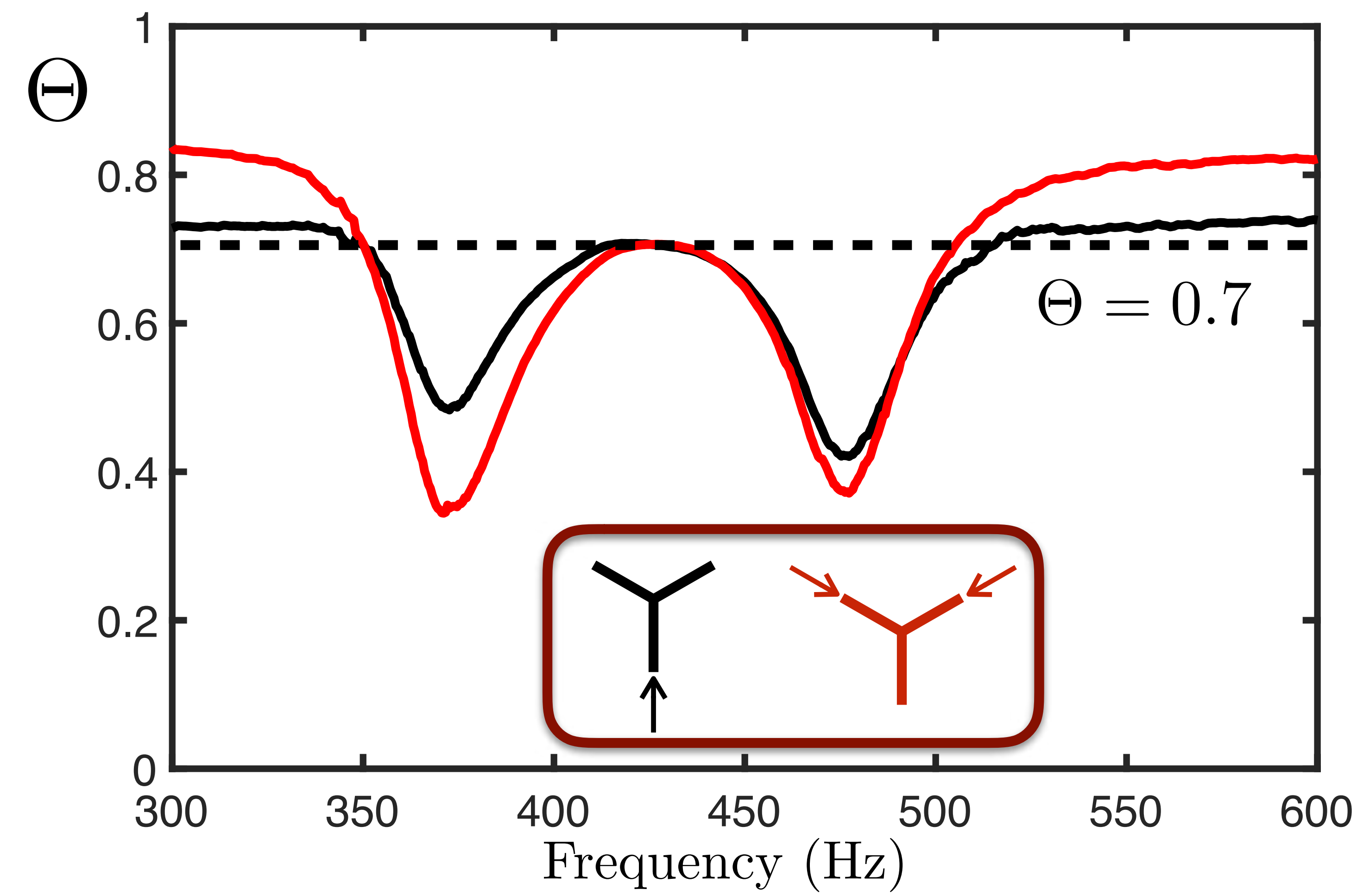




Geometry characteristics

$d = 5\,\text{cm}$

$\ell_{c1} = 2.9\,\text{cm}$

$\ell_{c2} = \ell_{c3} = 4.9\,\text{cm}$

subwavelength system

$\lambda/(2d) \approx 7.5$

losses

splitter combiner $\Theta(f^*) = 2|t(f^*)|^2$

perfect transmission $|t(f^*)| = 1/\sqrt{2} \approx 0.7$

Splitter and combiner with perfect transmission

Conditions for splitter and combiner with perfect transmission :

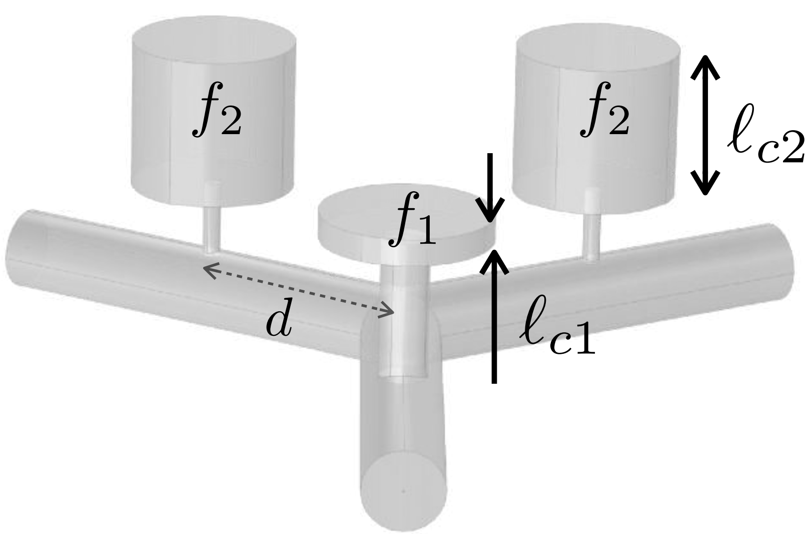
$$r(f^*) = 0$$
$$r'(f^*) + t'(f^*) = 0$$
$$|t(f^*)| = 1/\sqrt{2} \approx 0.7$$

$$\longrightarrow \quad |t'(f^*)| = |r'(f^*)| = 1/2$$

We use an optimization algorithm to achieve all the conditions and find the corresponding configurations (geometry)

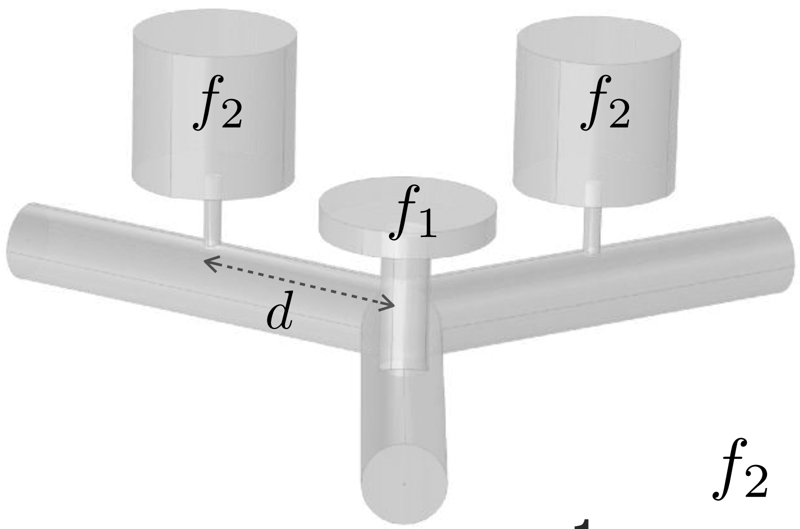
Operating frequency

$f^* = 200\,\mathrm{Hz}$



$$\ell_{c1} = 1.6\,\mathrm{cm}$$
$$\ell_{c2} = 9.4\,\mathrm{cm}$$
$$d = 13.9\,\mathrm{cm}$$

Splitter and combiner with perfect transmission



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Operating frequency

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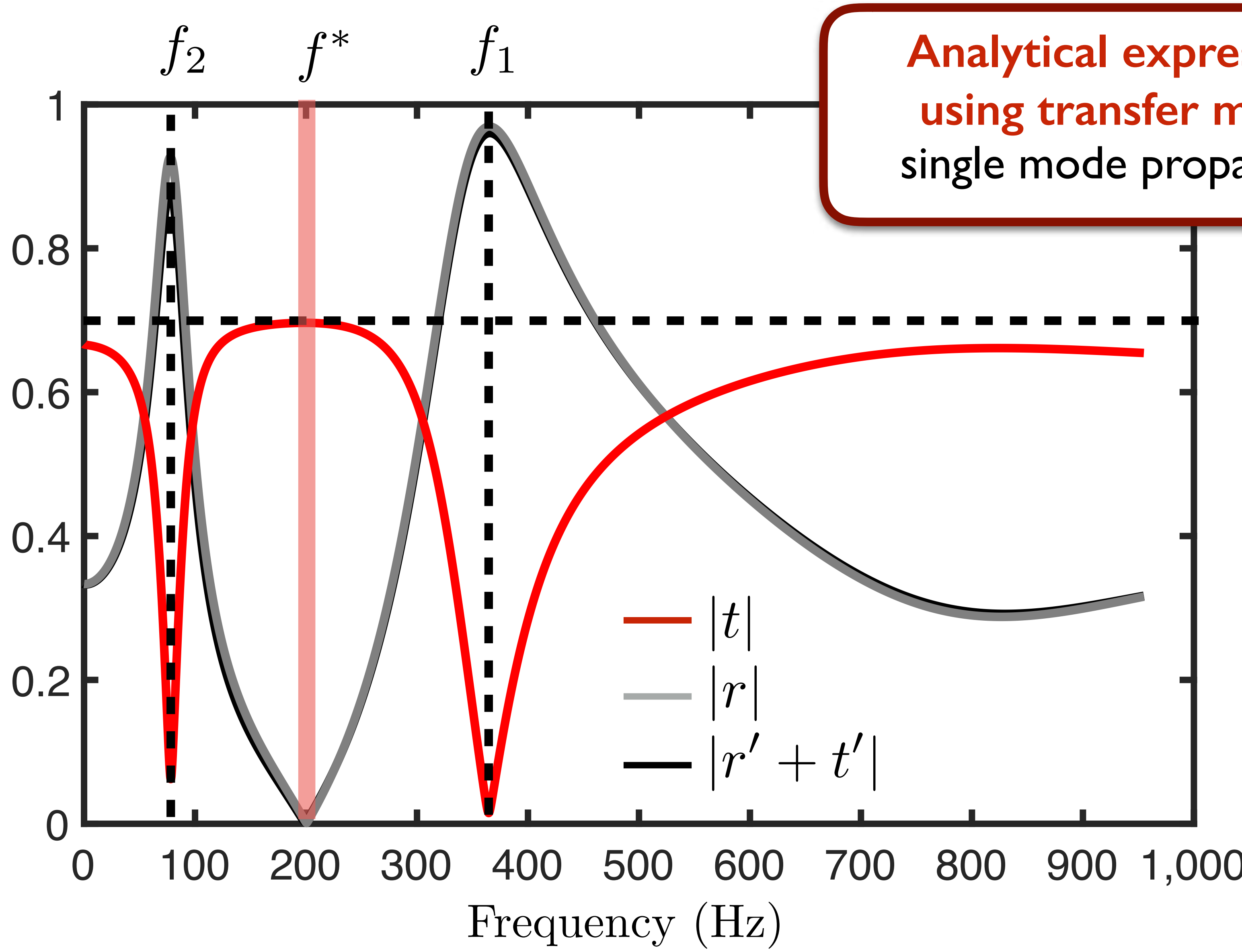
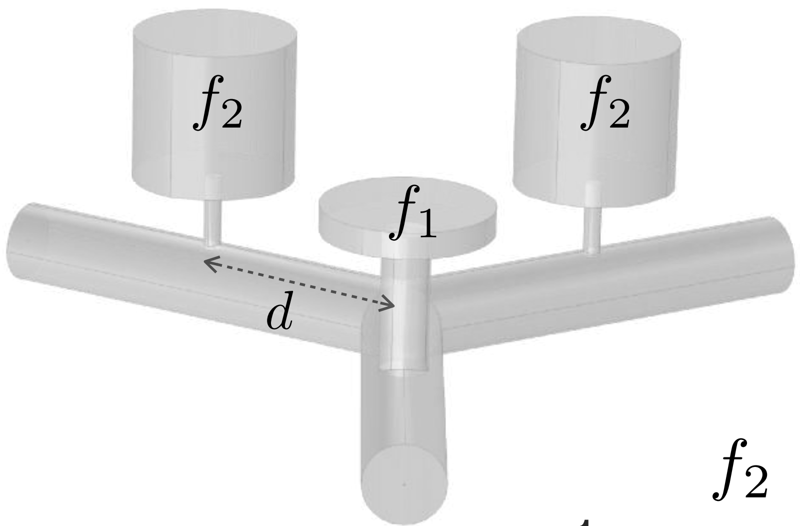


Analytical expressions using transfer matrix
single mode propagation

Splitter and combiner with perfect transmission



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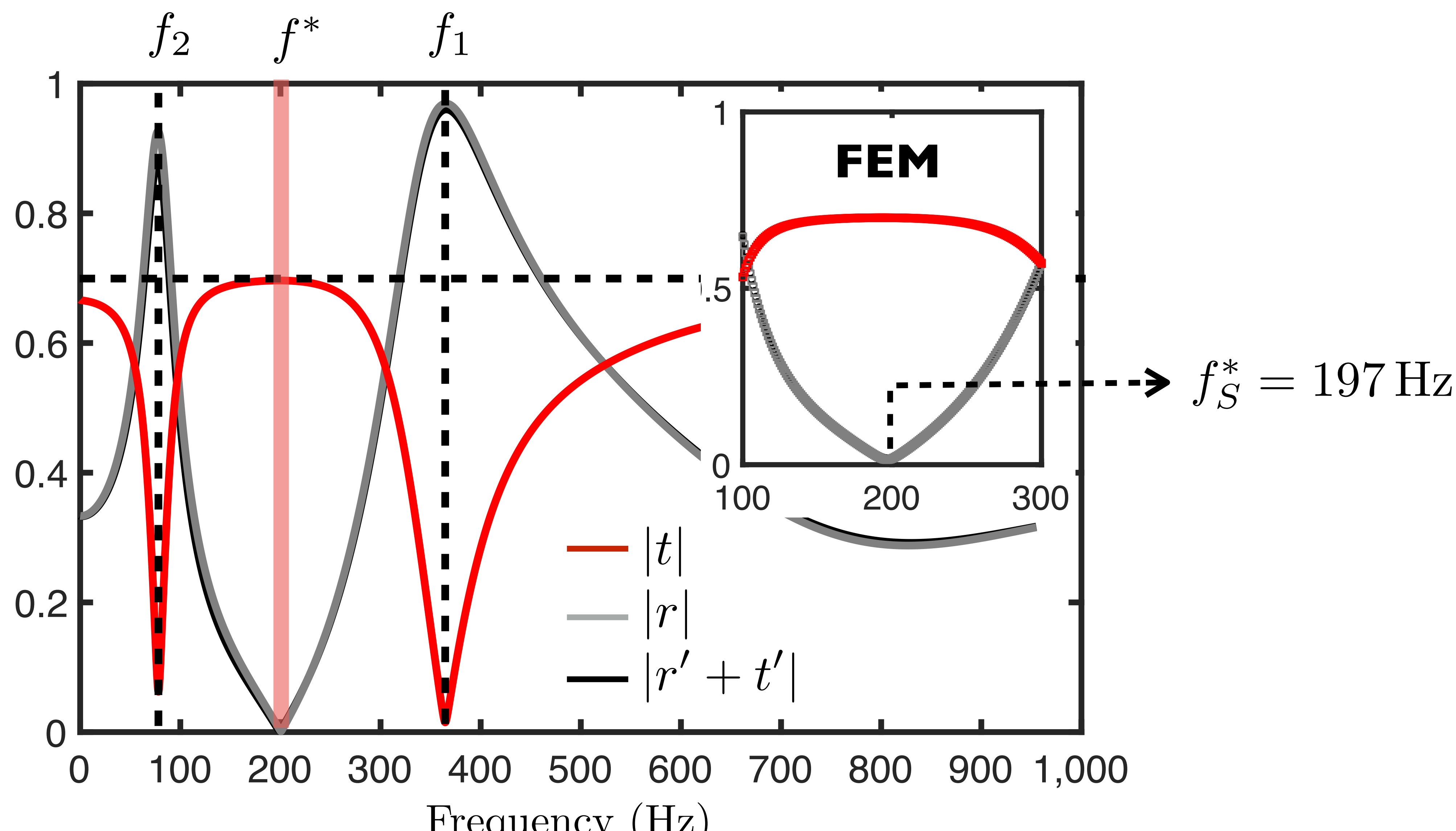
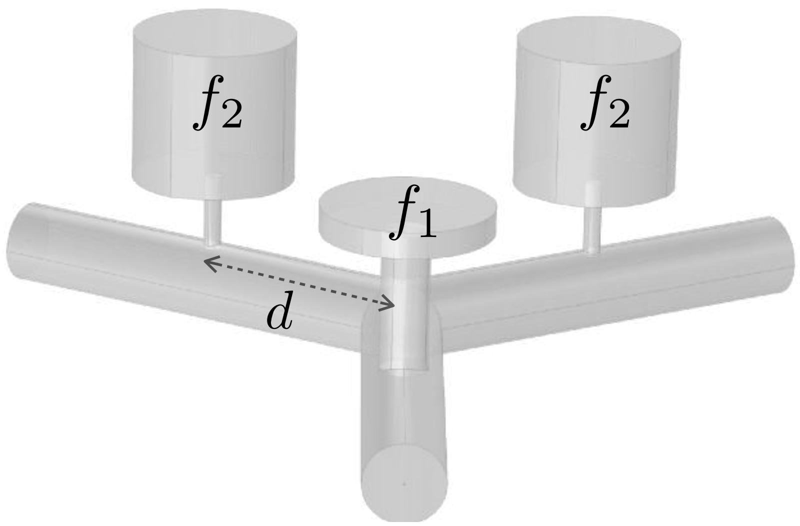


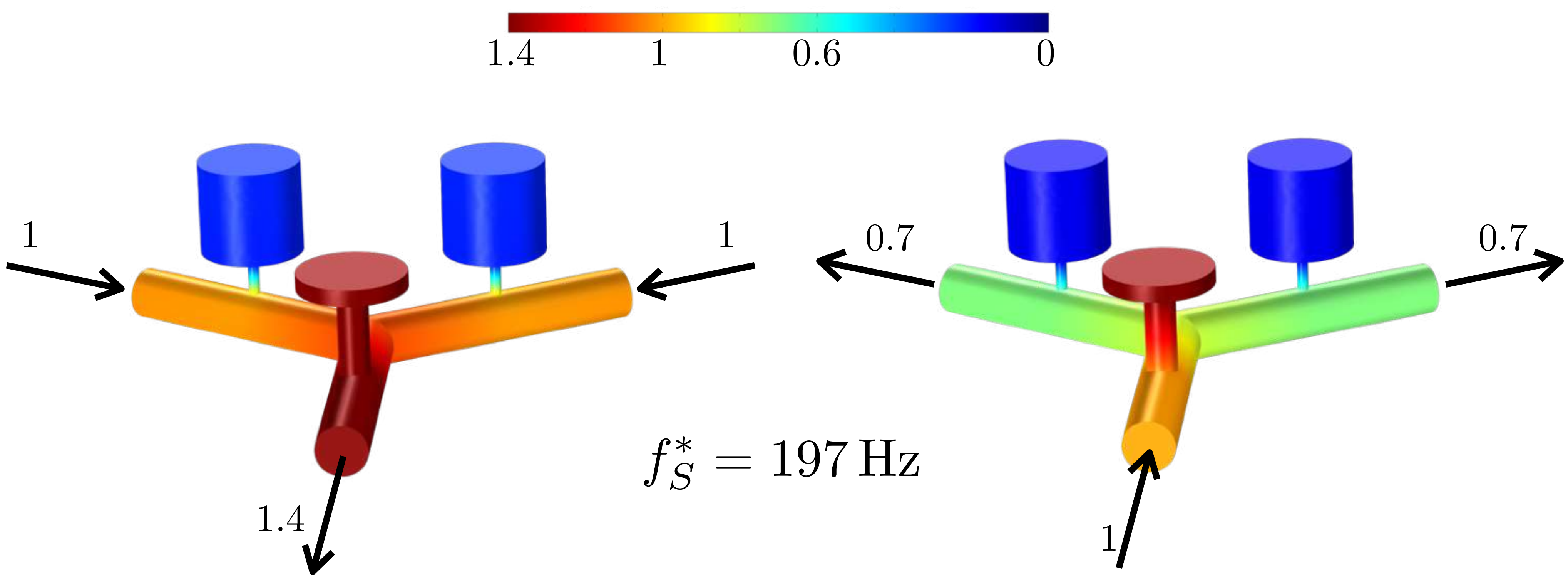
Splitter and combiner with perfect transmission



$$r(f^*) = 0$$
$$r'(f^*) + t'(f^*) = 0$$
$$|t(f^*)| = 1/\sqrt{2} \approx 0.7$$

Operating frequency

$$f_* = 200\,\text{Hz}$$

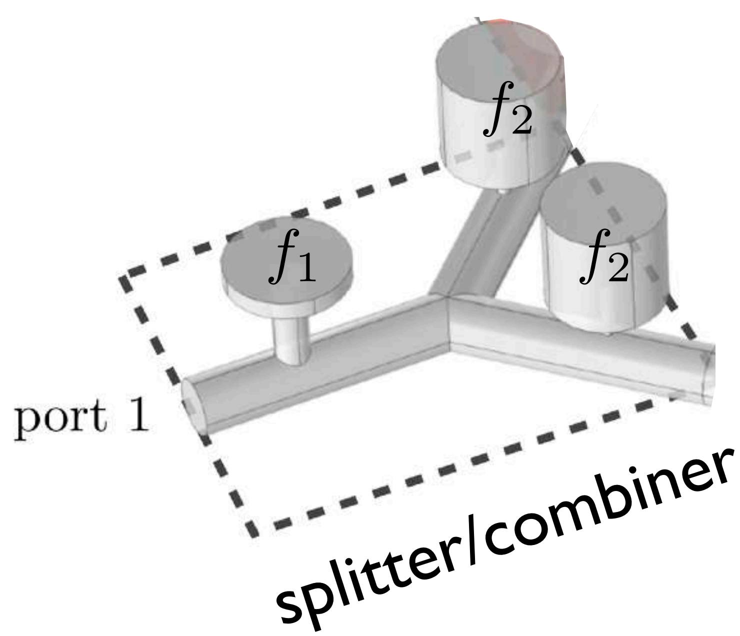


$$f_S^* = 197\,\text{Hz}$$

Combiner **Splitter**

3-port building blocks : towards complex networks

Optimized 3-port systems (splitter/combiner) can be used to design complex networks with <u>multiple waveguiding properties</u>



3-port building blocks : towards complex networks

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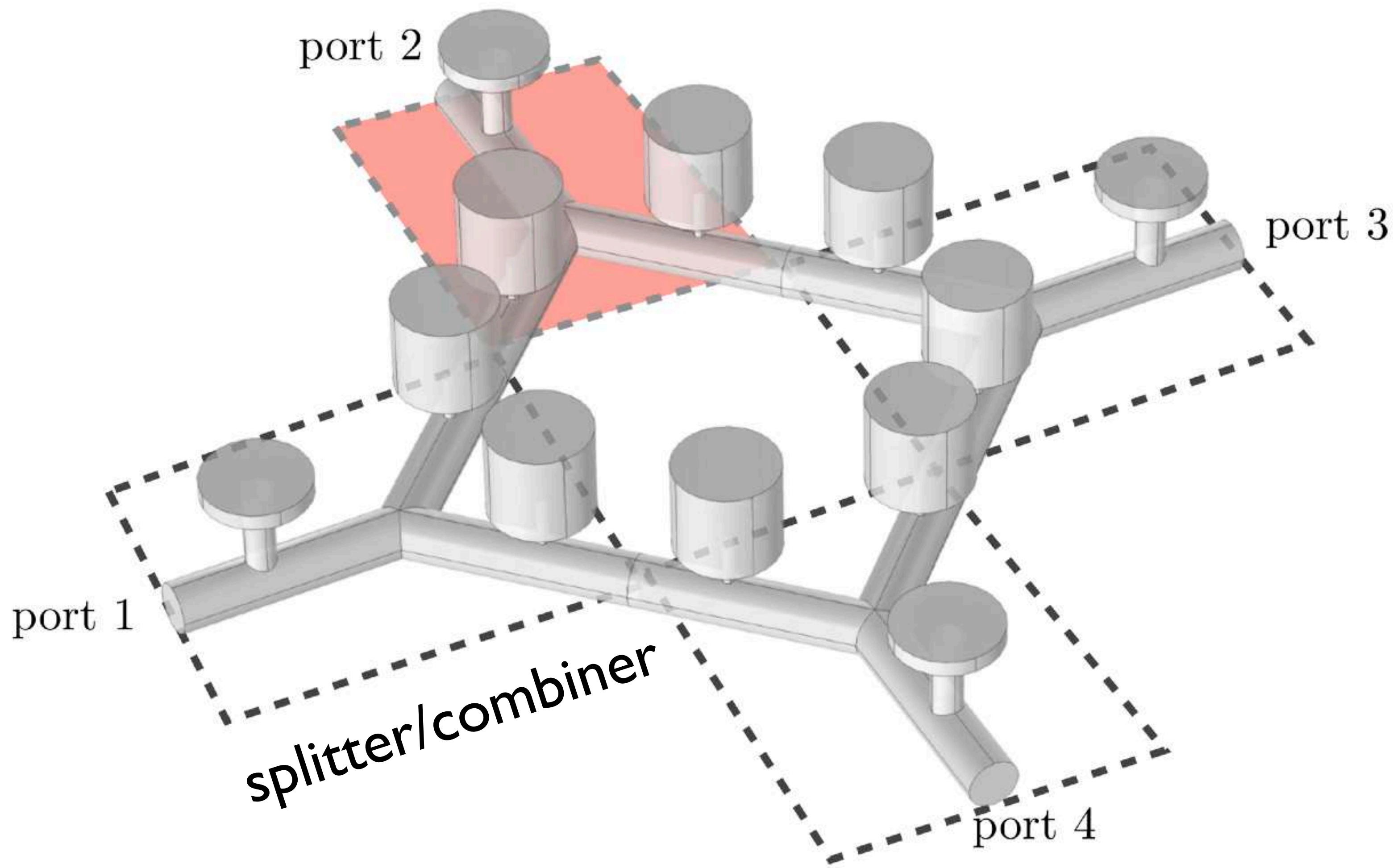

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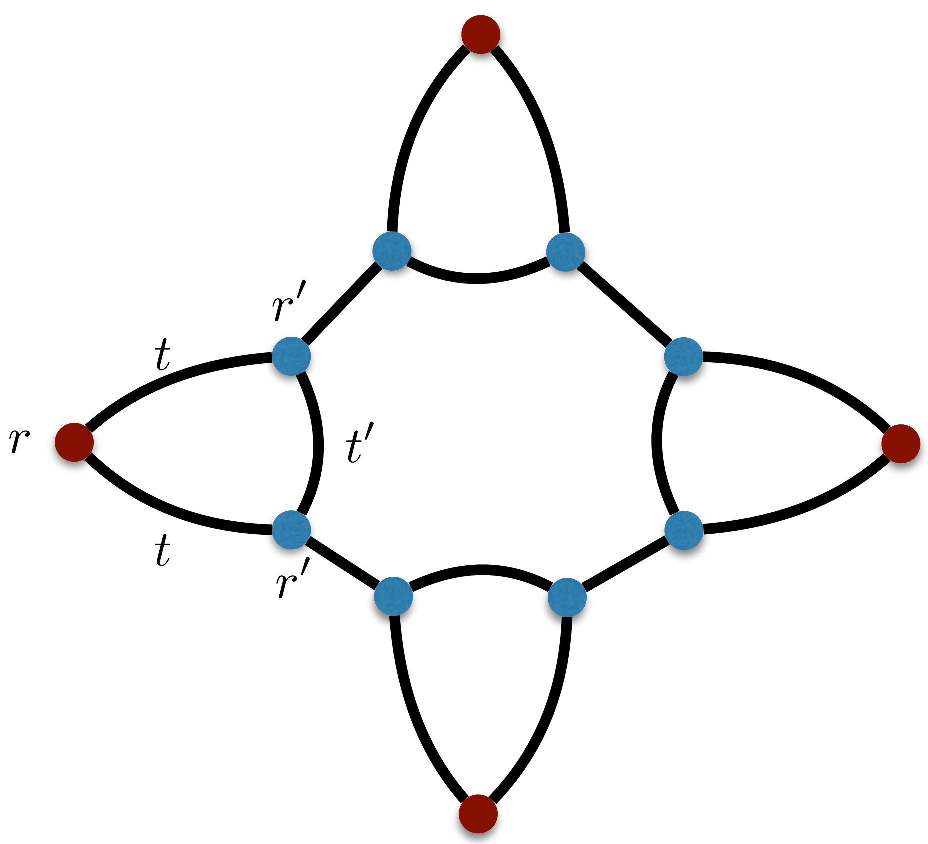

Scattering matrix

↓ star product

$$S = \begin{pmatrix} R & T_{12} & T_{13} & T_{14} \\ T_{12} & R & T_{23} & T_{24} \\ T_{13} & T_{23} & R & T_{34} \\ T_{14} & T_{24} & T_{34} & R \end{pmatrix}$$

with

$$R_i = r - t^2\left(\frac{1}{2p} - \frac{1}{2p'} + \frac{t' - r'}{P}\right)$$

$$T_{12} = T_{23} = T_{34} = T_{14} = -\frac{t^2}{pp'}$$

$$T_{13} = T_{24} = -\frac{2t^2t'}{pp'P}$$

$$p = 1 + r' + t', p' = -1 + r' + t' \text{ and } P = -1 + r'^2 + t'^2$$

3-port building blocks : towards complex networks

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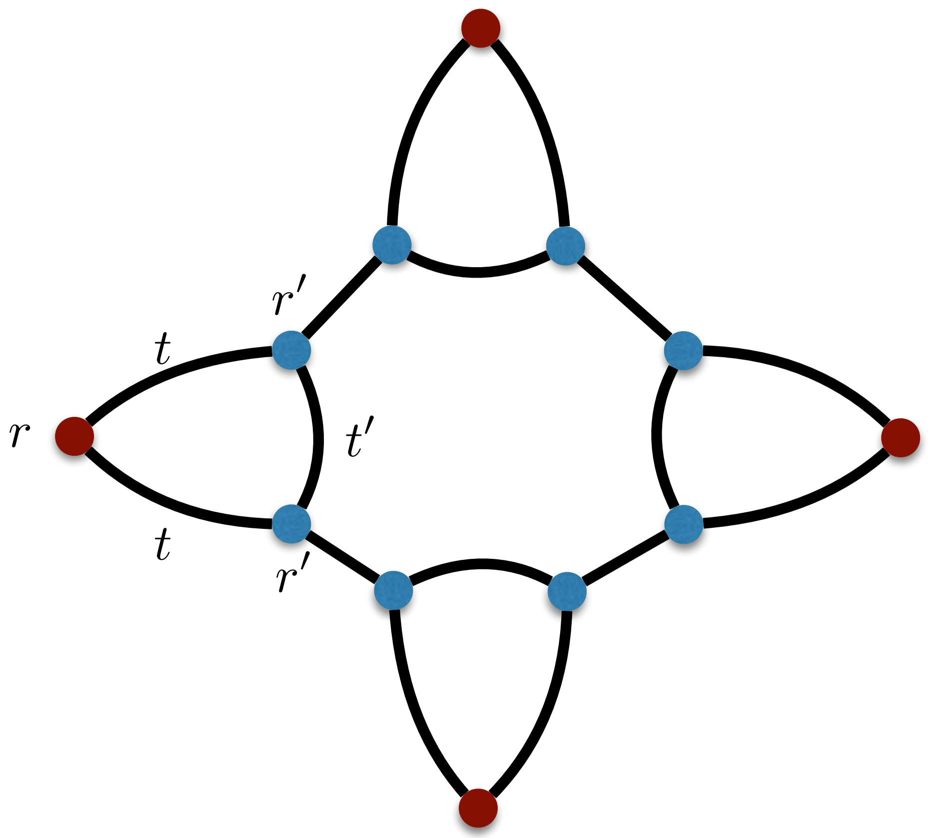

$$S = \frac{e^{2i\phi}}{2}\begin{pmatrix} -2t' & 1 & 2t' & 1 \\ 1 & -2t' & 1 & 2t' \\ 2t' & 1 & -2t' & 1 \\ 1 & 2t' & 1 & -2t' \end{pmatrix}$$

Scattering matrix

↓ star product

$$S = \begin{pmatrix} R & T_{12} & T_{13} & T_{14} \\ T_{12} & R & T_{23} & T_{24} \\ T_{13} & T_{23} & R & T_{34} \\ T_{14} & T_{24} & T_{34} & R \end{pmatrix}$$

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$$T_{12} = T_{23} = T_{34} = T_{14} = -\frac{t^2}{pp'}$$

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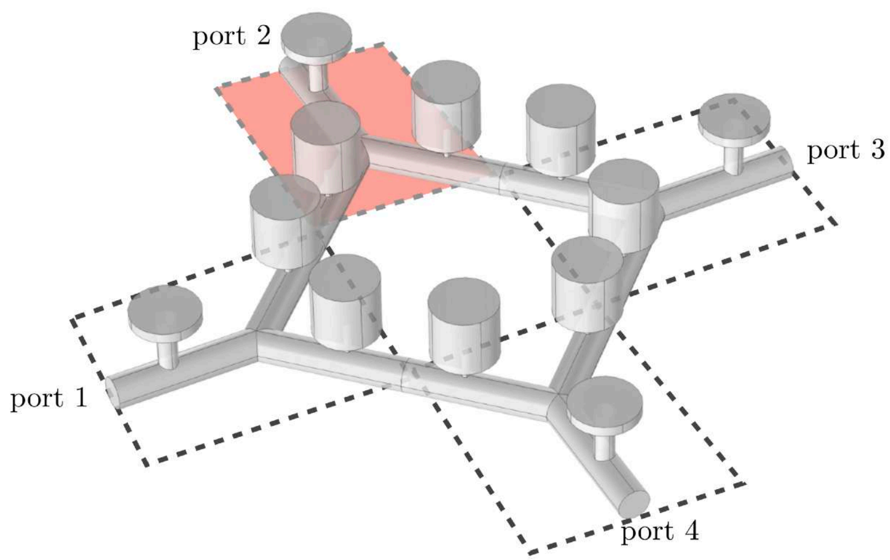
3-port building blocks : towards complex networks

Optimized 3-port systems (splitter/combiner) can be used to design complex networks with multiple waveguiding properties



Assuming splitter/combiner properties for each buiding block

↓

Scattering matrix

$$S = \frac{1}{2} e^{2i\phi} \begin{pmatrix} i & 1 & -i & 1 \\ 1 & i & 1 & -i \\ -i & 1 & i & 1 \\ 1 & -i & 1 & i \end{pmatrix}$$

ϕ : phase of t

Tuning of $t' = -i/2$

Operating frequency f^*

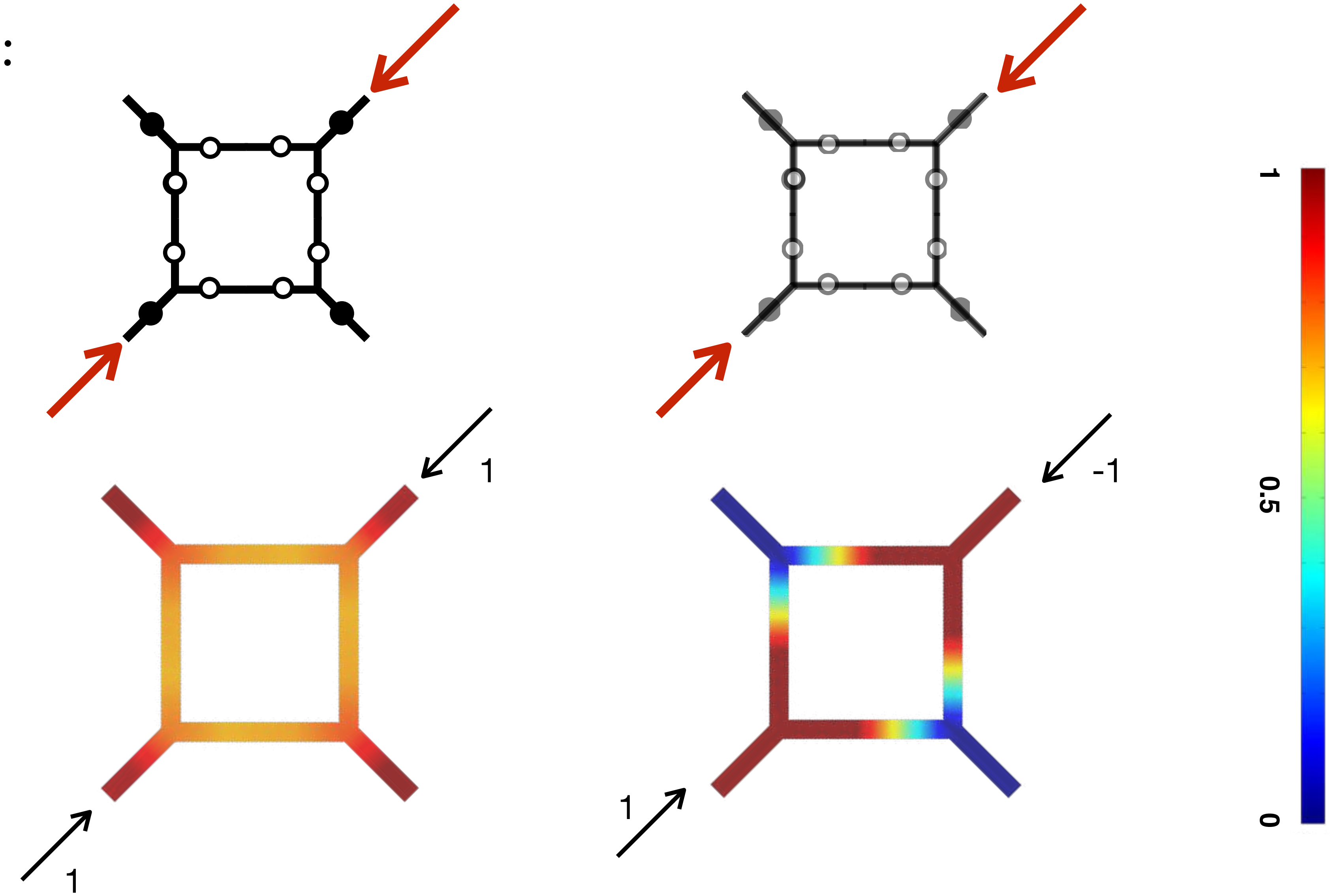
Wave routing depending on the phase of the input waves :

- orthogonal coupler
- transparency
- turn-right coupler
- turn-left coupler

4-port network for wave routing

Scattering from diagonal ports :

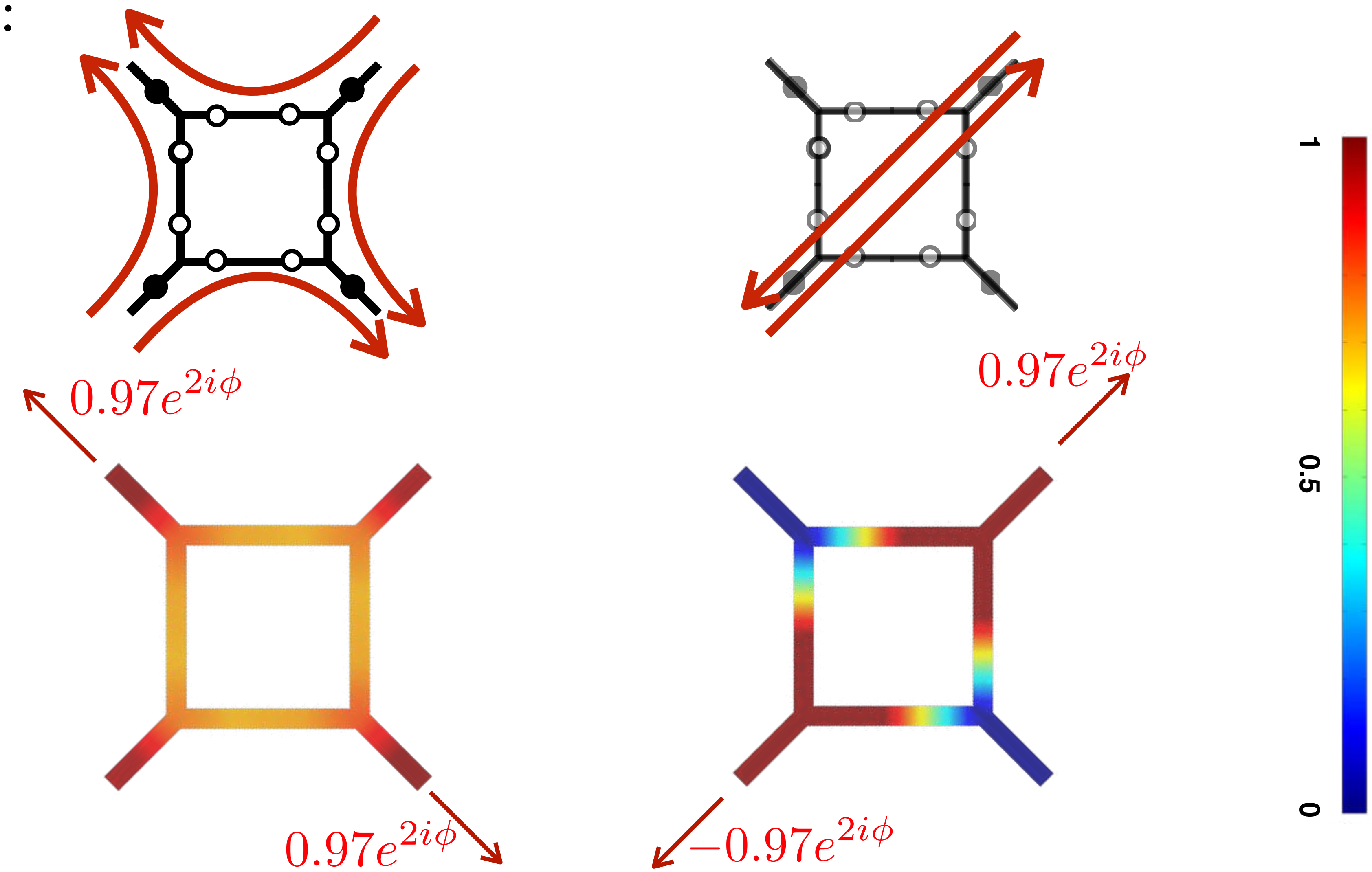
- Same amplitude
- Depending on phase



4-port network for wave routing

Scattering from diagonal ports :

- Same amplitude
- Depending on phase



$$\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}0\\1\\0\\1\end{pmatrix} e^{2i\phi}$$

Orthogonal coupler

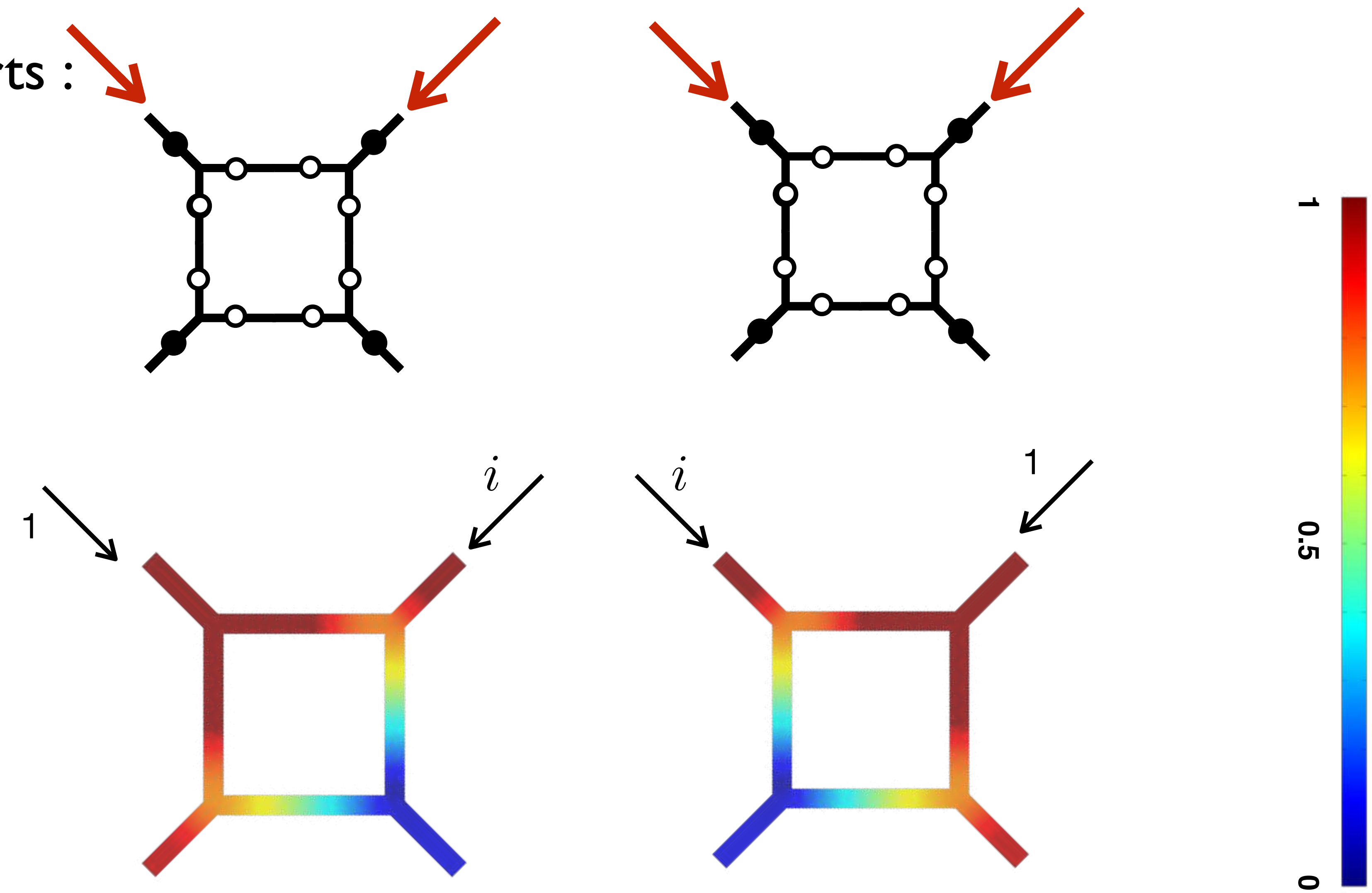
$$\begin{pmatrix}1\\0\\-1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}-1\\0\\1\\0\end{pmatrix} e^{2i\phi}$$

Transparency

4-port network for wave routing

Scattering from consecutive ports :

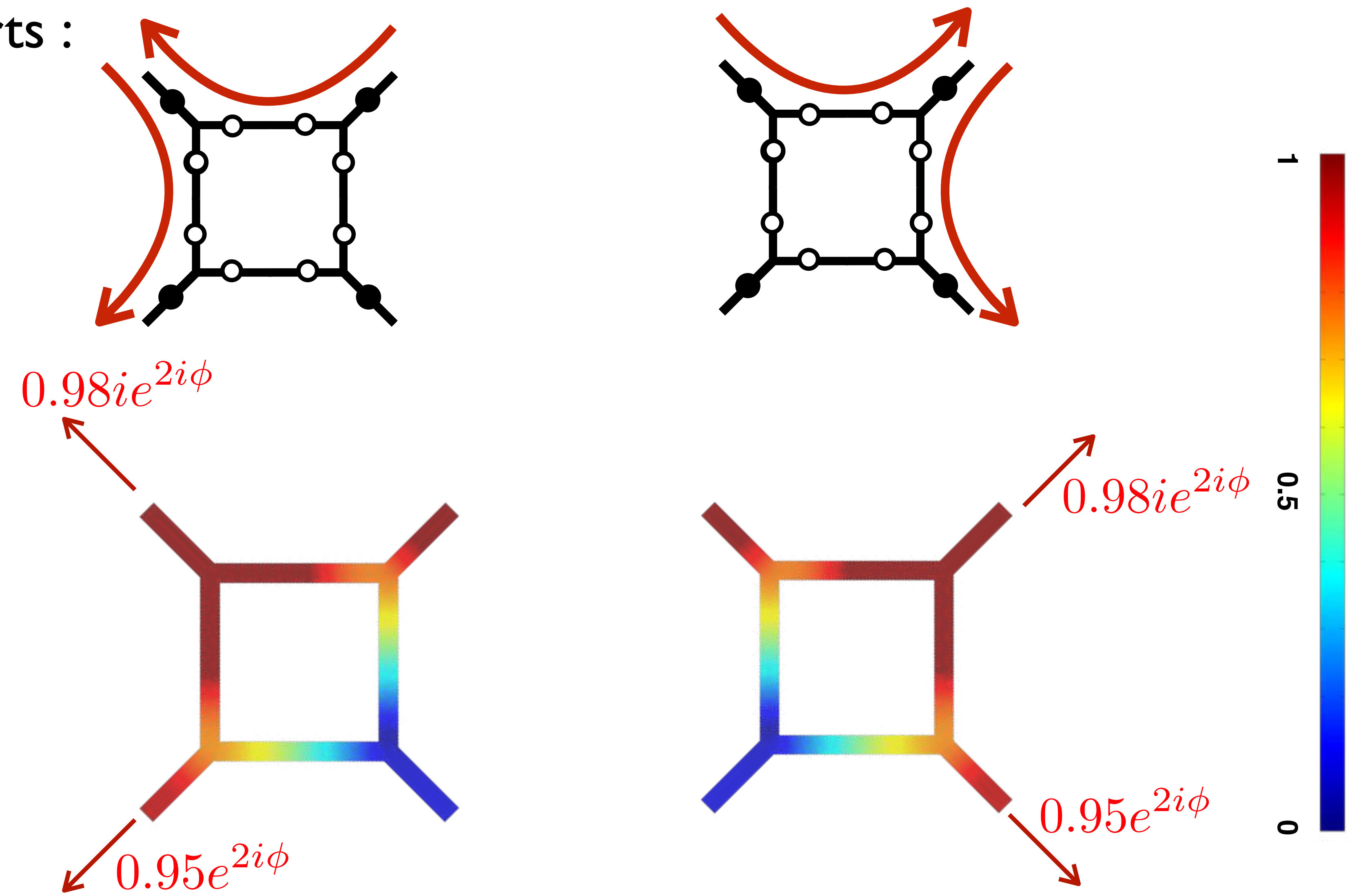
- Same amplitude
- Depending on phase



4-port network for wave routing

Scattering from consecutive ports :

- Same amplitude
- Depending on phase



$$\begin{pmatrix}0\\1\\i\\0\end{pmatrix} \longrightarrow \begin{pmatrix}1\\i\\0\\0\end{pmatrix} e^{2i\phi} \qquad \begin{pmatrix}0\\i\\1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}0\\0\\i\\1\end{pmatrix} e^{2i\phi}$$

Turn-right coupler

Turn-left coupler

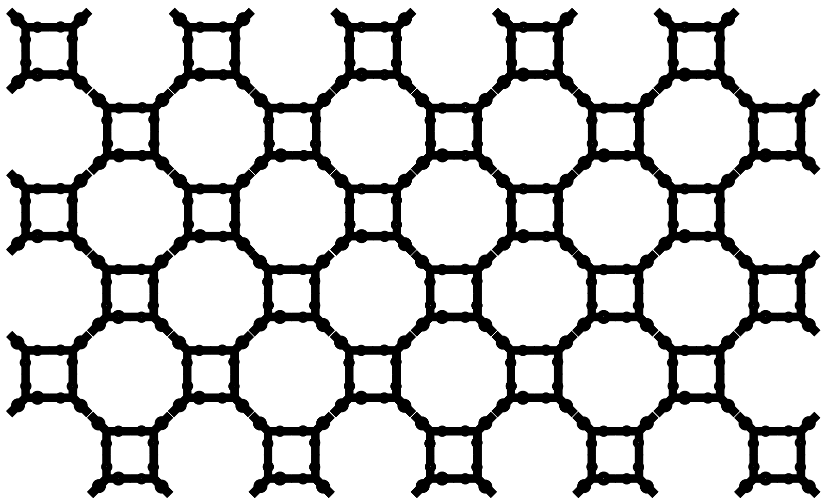
4-port network for wave routing

Conclusion

- Subwavelength splitter/combiner with perfect transmission
- 4-port network with 3-port building blocks
- Wave routing operations :
  - selection of the output channels with the relative phase of the inputs

Future work

- Tunable and compact system
- Change the orientation of the building blocks ⟶ other functionalities
- Use of more complex devices built from asymmetric 3-port
- Generalization to N-port systems
- Propagation in 2D network based on simple cells



Network