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GSRT
GENERAL SECRETARIAT FOR
RESEARCH AND TECHNOLOGY

State transfer in 1-D networks

"Workshop on local symmetries in wave physics"
Karystos, 4-6 September 2019

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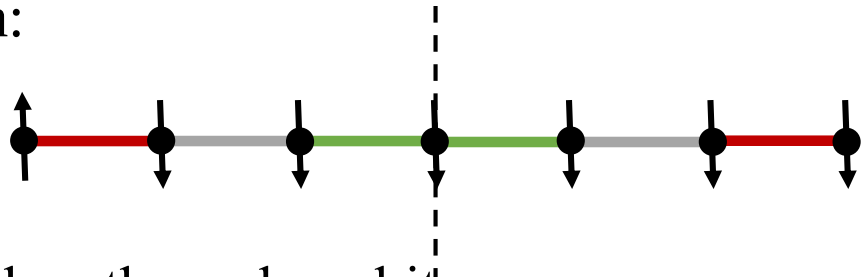
A 1-D Data Bus

Bose's idea:

$$H = J\sum_{i=1}^{N-1}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$$

Fidelity: $F = \left|\langle N|e^{-iHt}|1\rangle\right|^2$ Perfect state transfer (PST): $F = 1$

Christandl's idea:



Engineered J_i's: length can be arbitrary

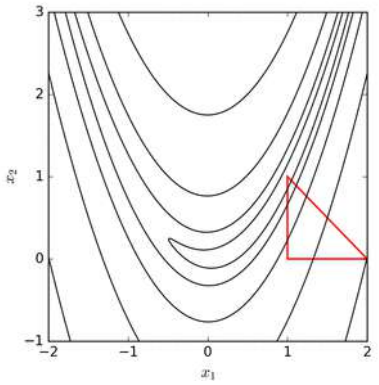
Optimization

Transfer for arbitrary initial and final sites, $|n\rangle \rightarrow |m\rangle$, for open and closed geometries:

$$F = \left|\langle m|e^{-iHt}|n\rangle\right|^2$$

In general: $F = f(J_1, \ldots, J_N, t)$

Nelder-Mead optimization algorithm



Reachability Criteria

Eigenvalue equation: $H_N v_i = E_i v_i$

$$H_N = \begin{pmatrix} 0 & J_1 & 0 & \cdots & & J_N \\ J_1 & 0 & J_2 & & & \\ 0 & J_2 & 0 & & & \\ \vdots & & & \ddots & & \\ & & & & J_{N-1} & \\ J_N & & & J_{N-1} & 0 & \end{pmatrix}$$

$$F = \left|\langle m|e^{-iHt}|n\rangle\right|^2 = \left|\sum_i^N v_{im} v_{in} e^{i\varphi_i}\right|^2, \text{ where } \varphi_i = E_i t$$

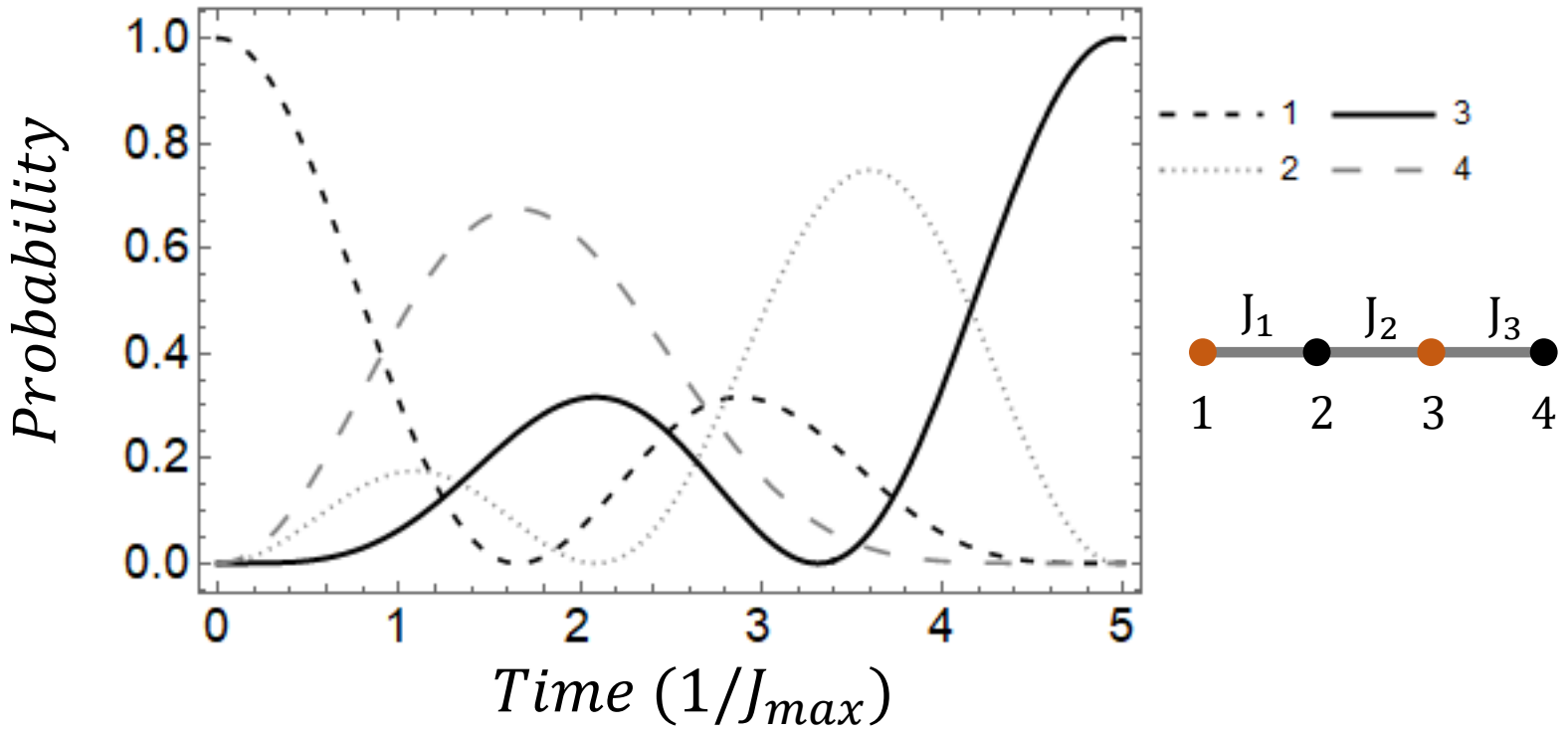
Necessary and sufficient conditions for PST

$$|v_{im}| = |v_{in}| \qquad \varphi_i = \begin{cases} n\pi \\ (2n+1)\frac{\pi}{2}, \end{cases} \quad n = 0,1,\ldots$$

Numerical Results

PST: $1 \rightarrow 3$

$$J_1 = J_{max}, \quad J_2 = 0.6J_{max}, \quad J_3 = 0.8J_{max}, \qquad t = 4.967$$



Non-reachable PST's

Open chains

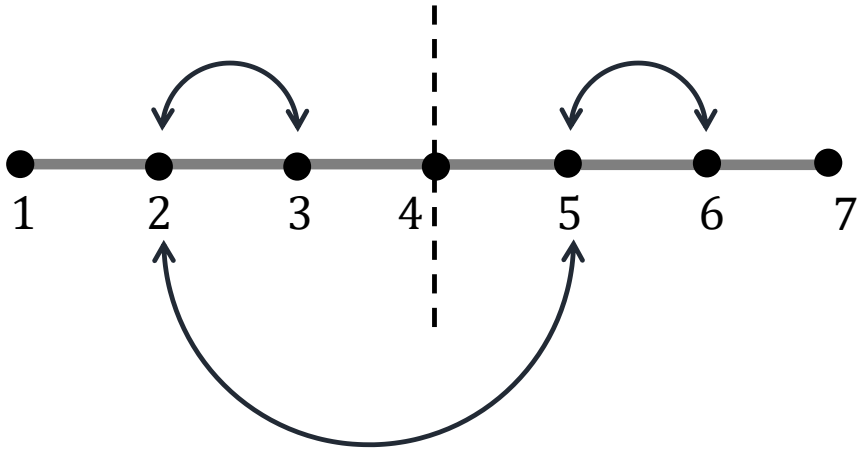
Even-sized chains:

$n \leftrightarrow m$, for $m > n$ and $m \leq N/2$

Odd-sized chains:

$n \leftrightarrow m$, for $m > n$ and $m \leq (N+1)/2$

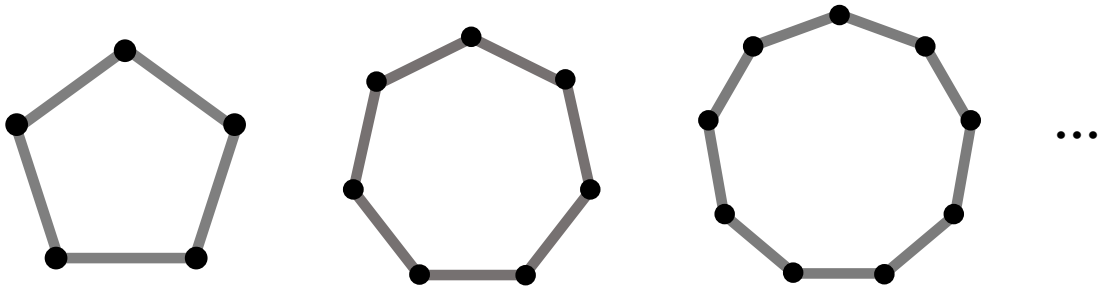
$n \leftrightarrow m$, when $n = even$ and $m = odd$



Closed chains

Odd-sized chains:

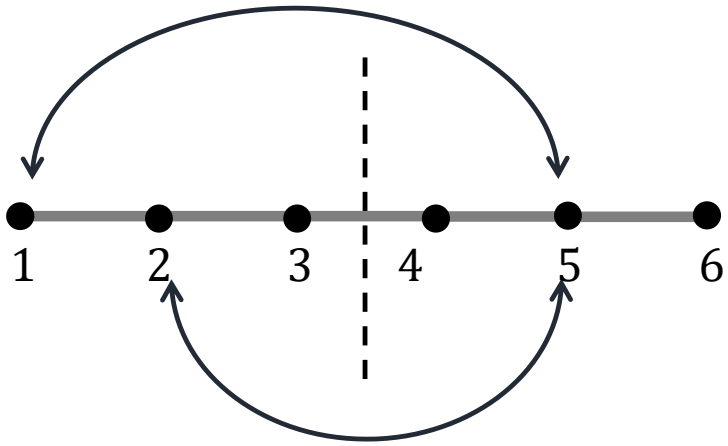
$n \leftrightarrow m$, for $N \neq 3$



Reachable PST's

Open chains

Mirror symmetric sites

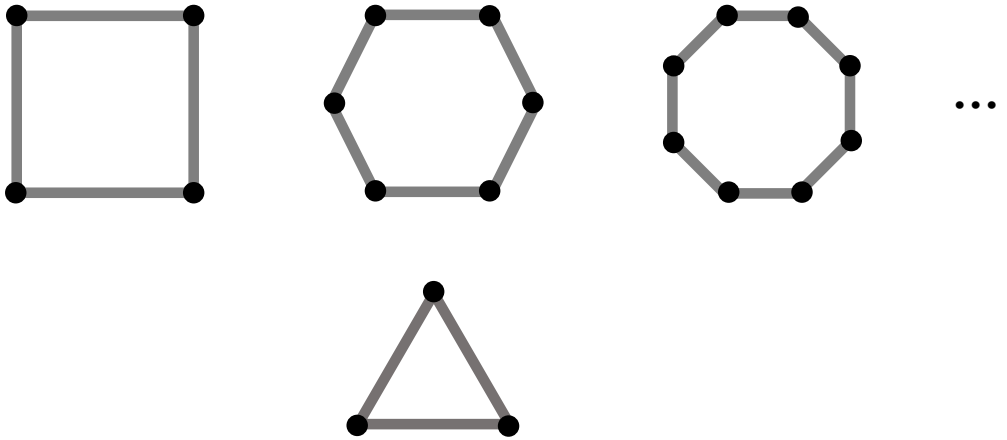


Even-sized chains:

$1 \leftrightarrow N - 1$

Closed chains

Even-sized chains

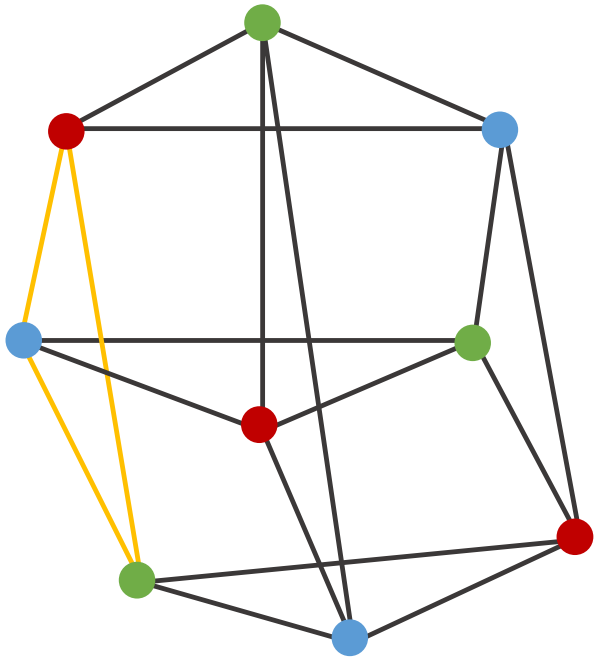


$n \leftrightarrow m$

$N = 3$ odd-sized chain

Perfect Graphs and Perfect State Transfer

Chromatic number = Maximal Clique number

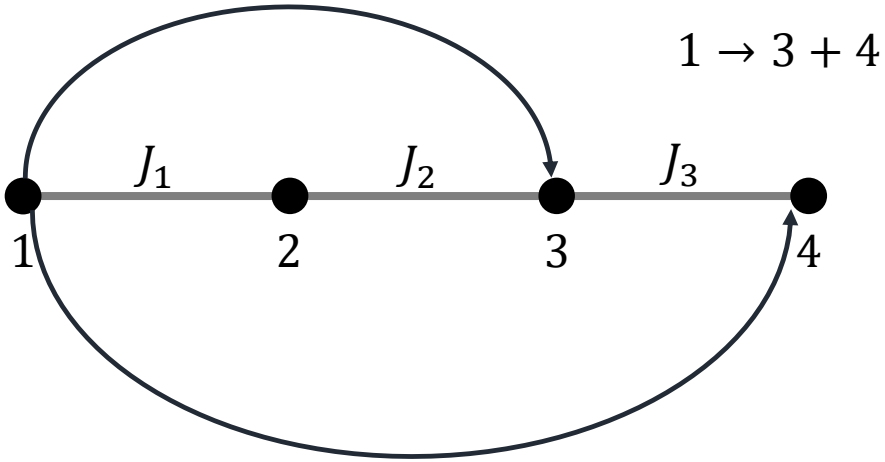


Conjecture:

"If a graph is not perfect, then PST is impossible between any pair of vertices"

Fractional Revival / Partial Transfer

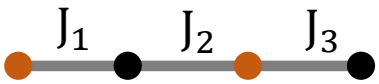
State Preparation & Generation of entanglement



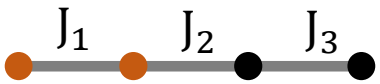
Derive sufficient and necessary conditions

Latent symmetries - Isospectral Matrix Reduction - PST

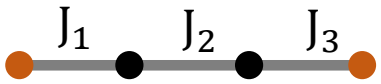
$$\begin{pmatrix} 0 & J_1 & 0 & 0 \\ J_1 & 0 & J_2 & 0 \\ 0 & J_2 & 0 & J_3 \\ 0 & 0 & J_3 & 0 \end{pmatrix}$$



$$\begin{pmatrix} \frac{J_1{}^2}{\lambda} & \frac{J_1 J_2}{\lambda} \\ \frac{J_1 J_2}{\lambda} & \frac{J_2{}^2}{\lambda} + \frac{J_3{}^2}{\lambda} \end{pmatrix}$$

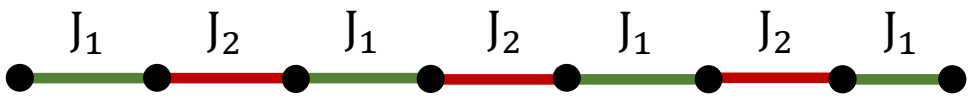


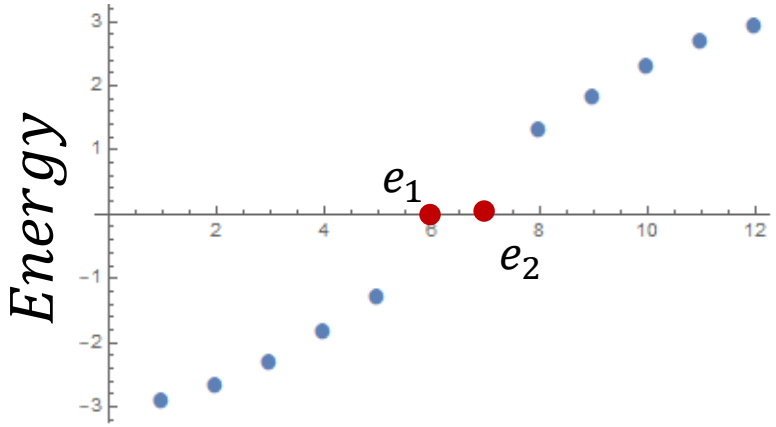
$$\begin{pmatrix} 0 & J_1 \\ J_1 & \frac{\lambda^2 J_2}{\lambda^2 - J_3{}^2} \end{pmatrix}$$

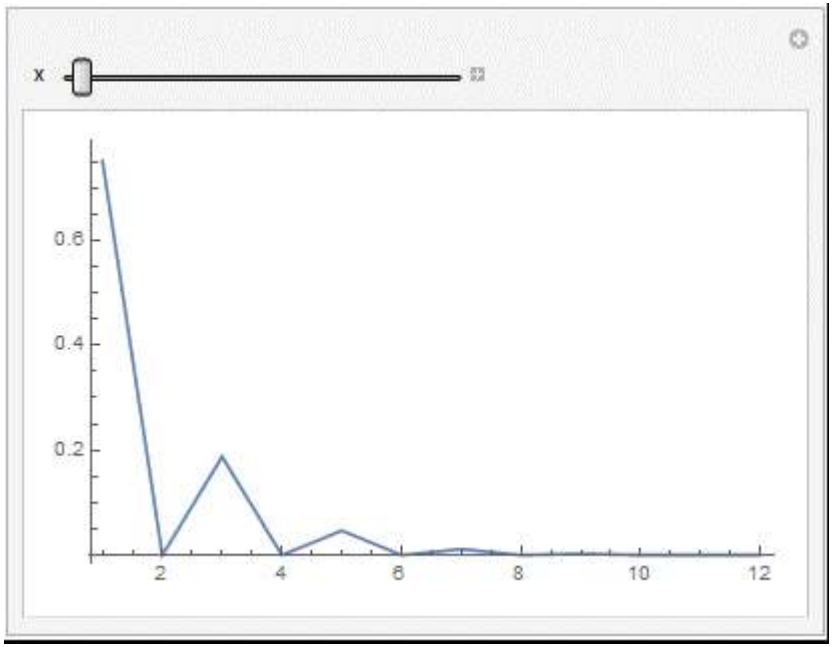


$$\begin{pmatrix} \frac{J_1{}^2 \lambda}{\lambda^2 - J_2{}^2} & \frac{J_1 J_2 J_3}{\lambda^2 - J_2{}^2} \\ \frac{J_1 J_2 J_3}{\lambda^2 - J_2{}^2} & \frac{J_3{}^2 \lambda}{\lambda^2 - J_2{}^2} \end{pmatrix}$$

State transfer protocol – SSH model



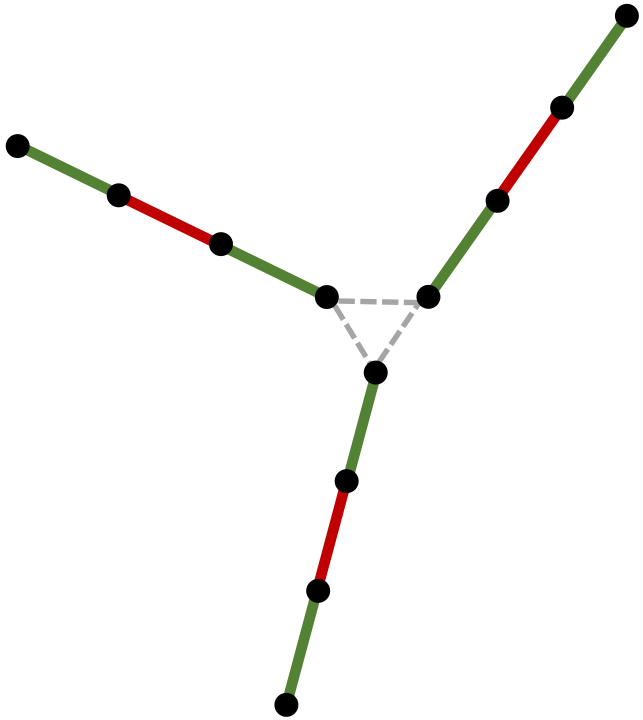




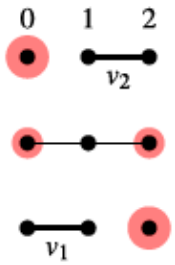
Initial state: $\frac{1}{\sqrt{2}}(|e_1\rangle - |e_2\rangle)$

Target state: $\frac{1}{\sqrt{2}}(|e_1\rangle + |e_2\rangle)$

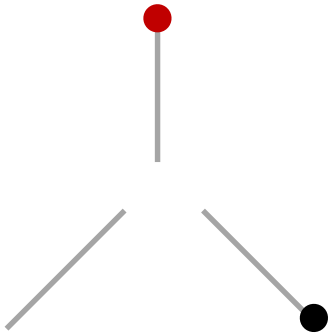
SSH Tri-junction



Boros et al. (2019)

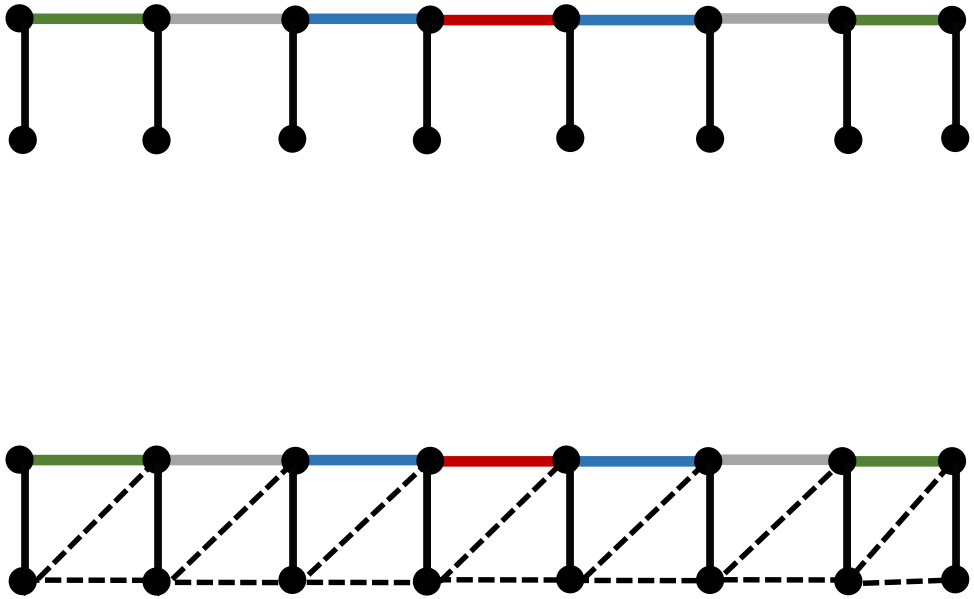


Y gate: $-i\sigma_y$



We may need an extra leg!

Introducing environment



Thank you!

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