Optimal topological state transfer

WORKSHOP ON LOCAL SYMMETRIES
IN WAVE PHYSICS

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What we actually promise to do: Time-Optimal non-adiabatic Thouless pumping

- Adiabatic time modulation of 1D potential parameters mimics 2D topological Chern insulator
- Integer Chern number in one cycle of time-evolution corresponds to integer number of charges pumped from one end to the other

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Quantization of particle transport

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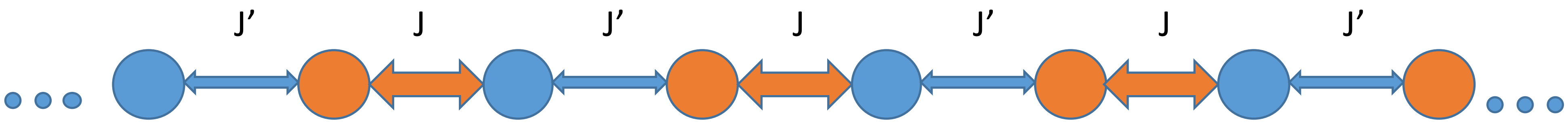
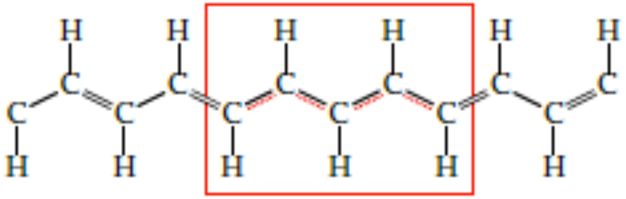
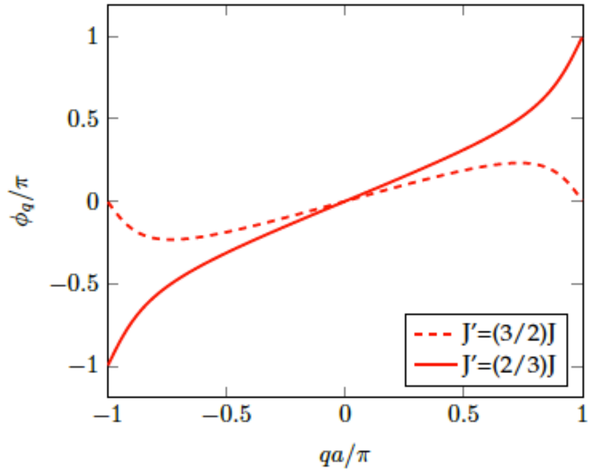
The integrated particle current produced by a slow periodic variation of the potential of a Schrödinger equation is evaluated. It is shown that in a finite torus the integral of the current over a period can vary continuously, but in an infinite periodic system with full bands it must have an integer value. This quantization of particle transport is used to classify the energy gaps in a one-dimensional system with competing or incommensurate periods. It is also used to rederive Prange's results for the fractional charge of a soliton.

What we actually intend to do: Time-Optimal edge state transfer in SSH chain

Su-Schriefer-Heeger model (1979-80) of polyacetilyne (1D solitons)





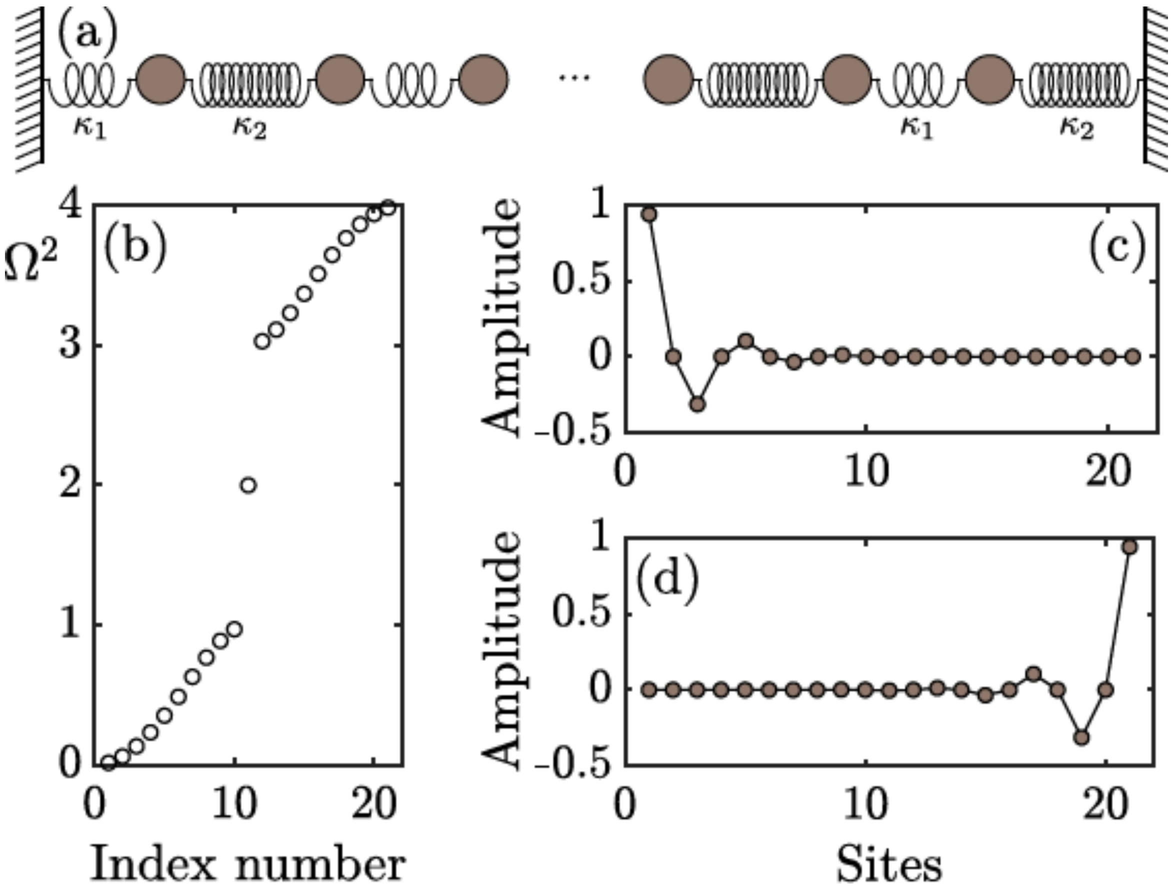


Zak-phase topological order from Dynamical matrix.

Topologically trivial: ϕ=0 non-trivial ϕ=1

$$|u_q^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \mp e^{i\phi_q}\end{pmatrix} \qquad \mathcal{A}_q^{(\pm)} = i\,\langle u_q^{(\pm)}|\partial_q u_q^{(\pm)}\rangle = -\frac{1}{2}\frac{d\phi_q}{dq} \qquad \Phi_{\text{Zak}} = \int_{-\pi/a}^{+\pi/a} \mathcal{A}_q^{(\pm)}\,dq = -\frac{1}{2}\int_{-\pi/a}^{+\pi/a} \frac{d\phi_q}{dq}\,dq.$$

What we actually started with:
Time-optimal edge state transfer in **mechanical** SSH chain



$$\hat{D}(q) = \begin{pmatrix} \frac{\kappa_1+\kappa_2}{m} & -\frac{\kappa_2+\kappa_1 e^{-iqa}}{m} \\ -\frac{\kappa_2+\kappa_1 e^{iqa}}{m} & \frac{\kappa_1+\kappa_2}{m} \end{pmatrix}$$

The equations of motion for this system read:

$$\frac{d^2 q_n^{(1,2)}}{dt^2} = \mp k_1 (q_{n,n+1}^{(1)} - q_{n-1,n}^{(2)}) \pm k_2 (q_n^{(2)} - q_n^{(1)})$$

for particles (1,2) corresponding to each unit cell as depicted in Fig. 1(a) with fixed boundary conditions $q_0^{(1)} = q_{N+1}^{(2)} = 0$.[9] Time t scales as $\sqrt{m/k_0}$, stiffness k_1 and k_2 with k_0, and amplitudes q with normalization length L of the edge state amplitude vector: $L = \sqrt{\sum_n \tilde{q}_{1/2,n}^2}$, where amplitude vector $\tilde{\mathrm{q}}$ is derived from $-\tilde{\omega}^2 \tilde{\mathrm{q}} = \mathbf{K}\tilde{\mathrm{q}}$ with[10]

$$\mathbf{K} = \begin{bmatrix} -\overline{k} & k_1 & 0 & 0 & \dots \\ k_1 & -\overline{k} & k_2 & 0 & \dots \\ 0 & k_2 & -\overline{k} & k_1 & \dots \\ 0 & 0 & k_1 & -\overline{k} & k_2 \end{bmatrix}$$

where $\overline{k} = k_1 + k_2$.

We do it! Both in mechanical and quantum

Mechanical

21 masses (no other odd case in literature)

k_1-=3 k_2=1 fidelity >99%

Cos (like Mei 2018): T=300

Linear: T=195

Tangential: T=146

Symmetric optimal: T=40

Assymetric optimal: T=15!

If you can tune initial phase

T->1!!!

Quantum

Longhi (2019): CTAP case 31 sites >90% fidelity

J_1=1 J_2=0

Minimum times for this case:

Mei (2018): T=800

Longhi (2019): T=200

WE (2019): T->2!!!

Lang&Buechler (2017) even case N=10

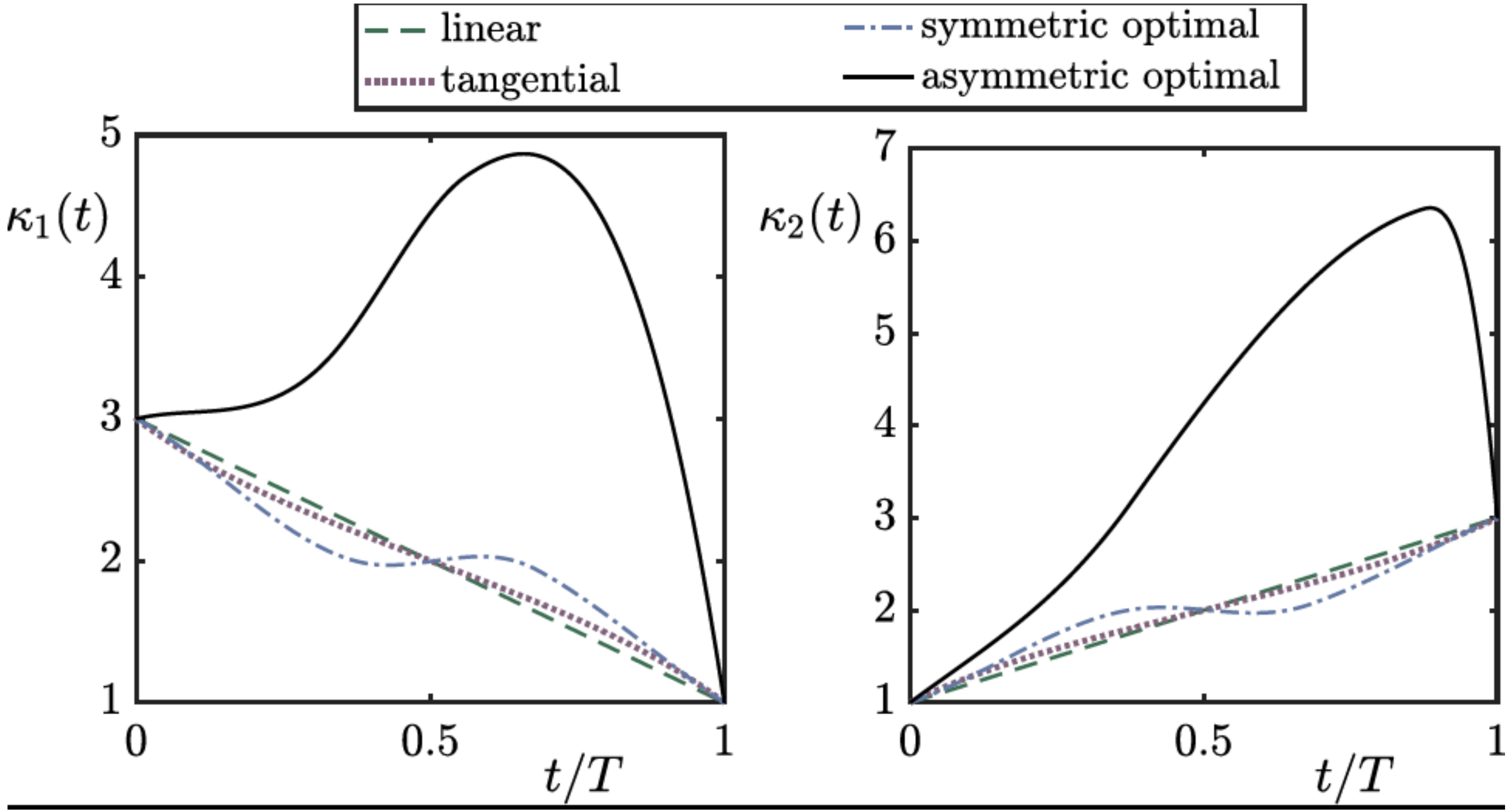
VERY DIFFERENT FROM ODD

One parameter change T=40

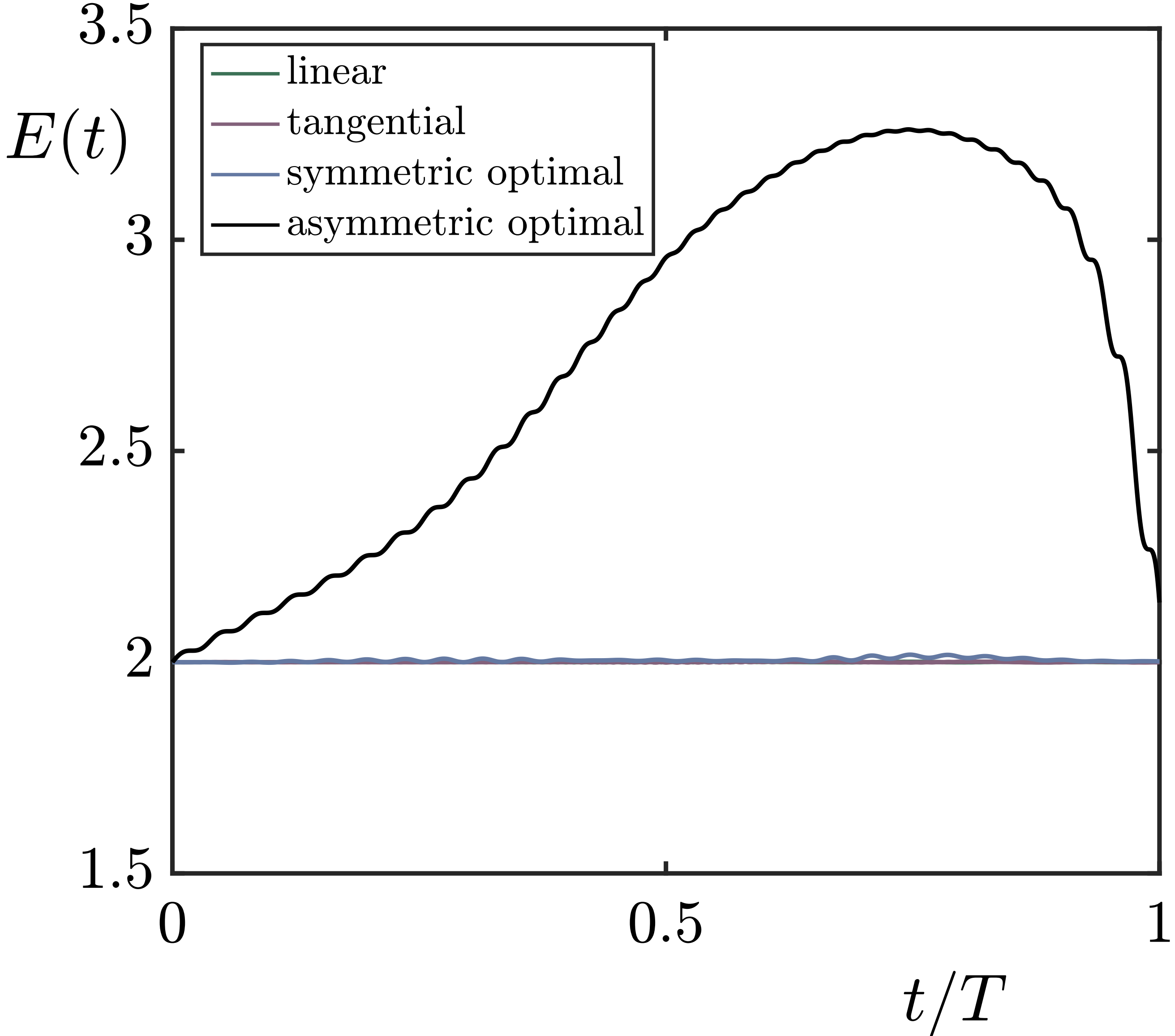
Buchler pol/sin: <30% Fidelity

WE 99.9% fidelity crossing gap

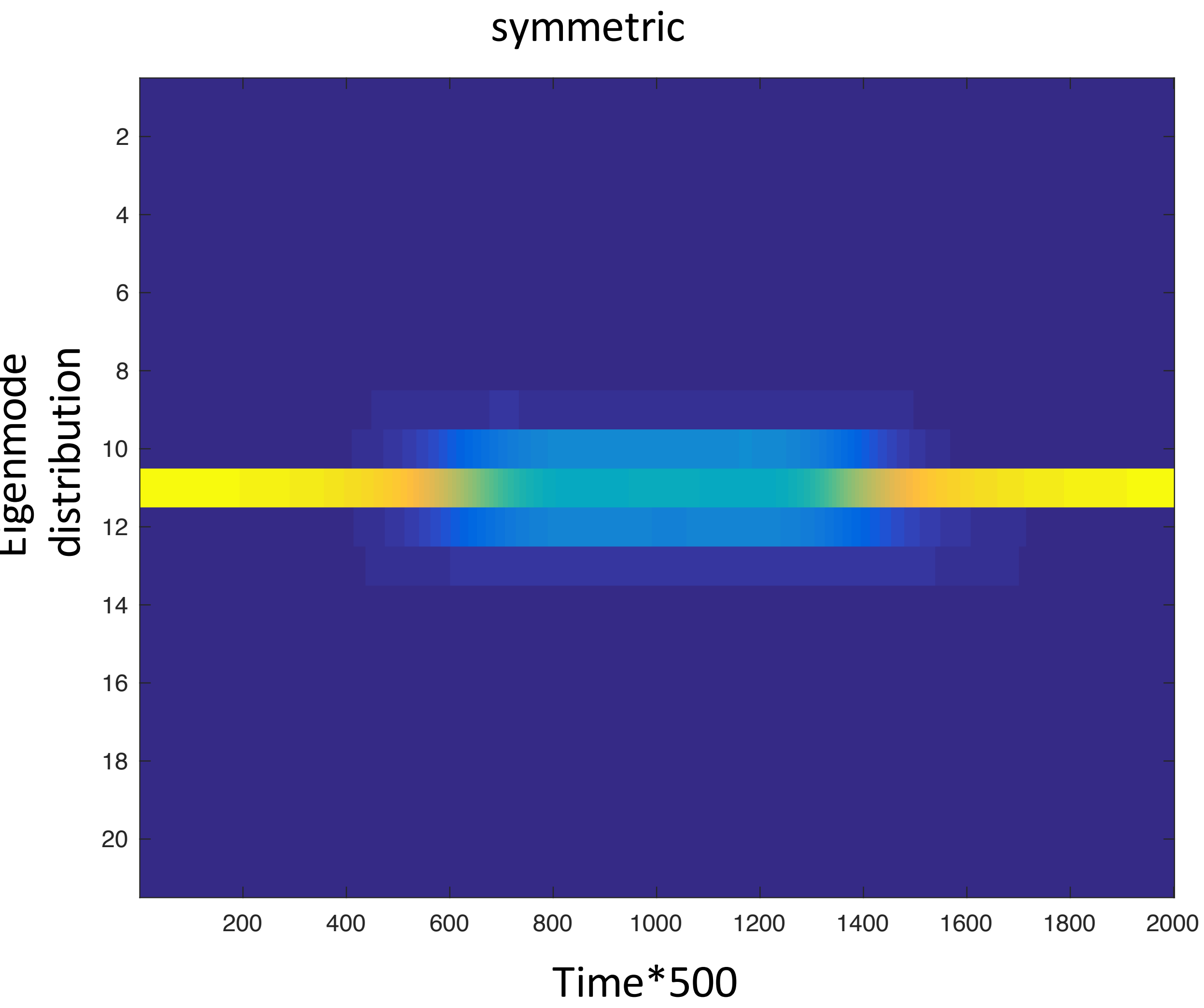
BUT we have ->0 clue
CLUE 1: The form of the control functions

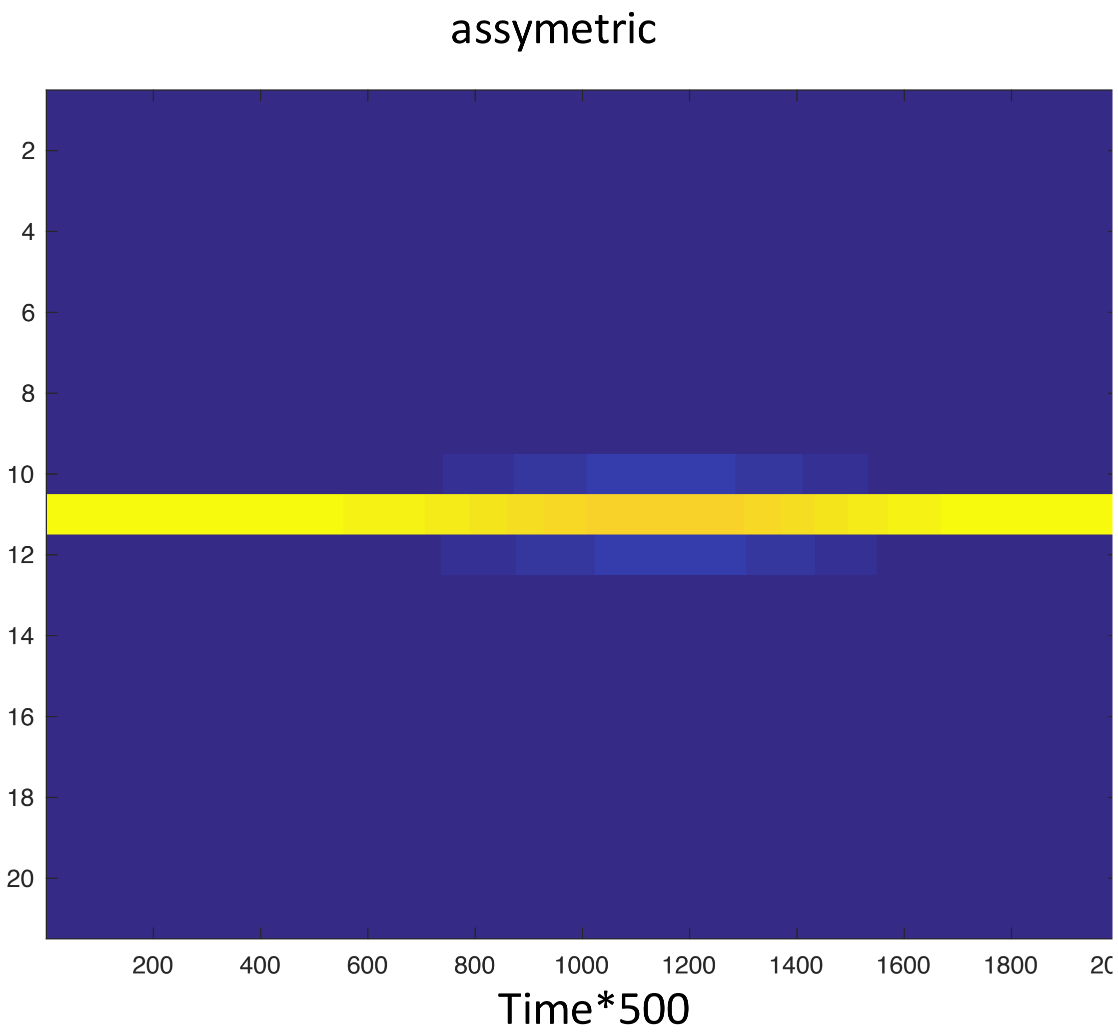


BUT we have ->0 clue
CLUE 2: Energy Exchange



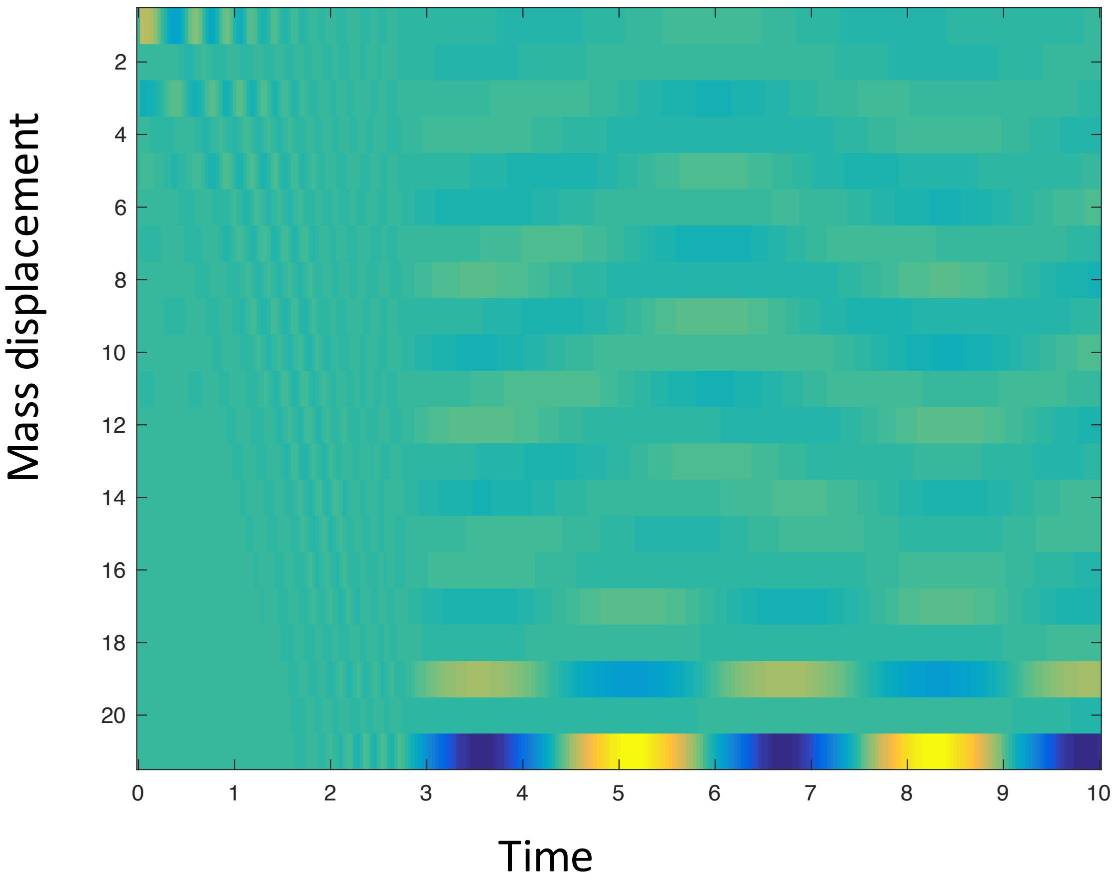
BUT we have ->0 clue
CLUE 3: Instantaneous Bulk Excitation

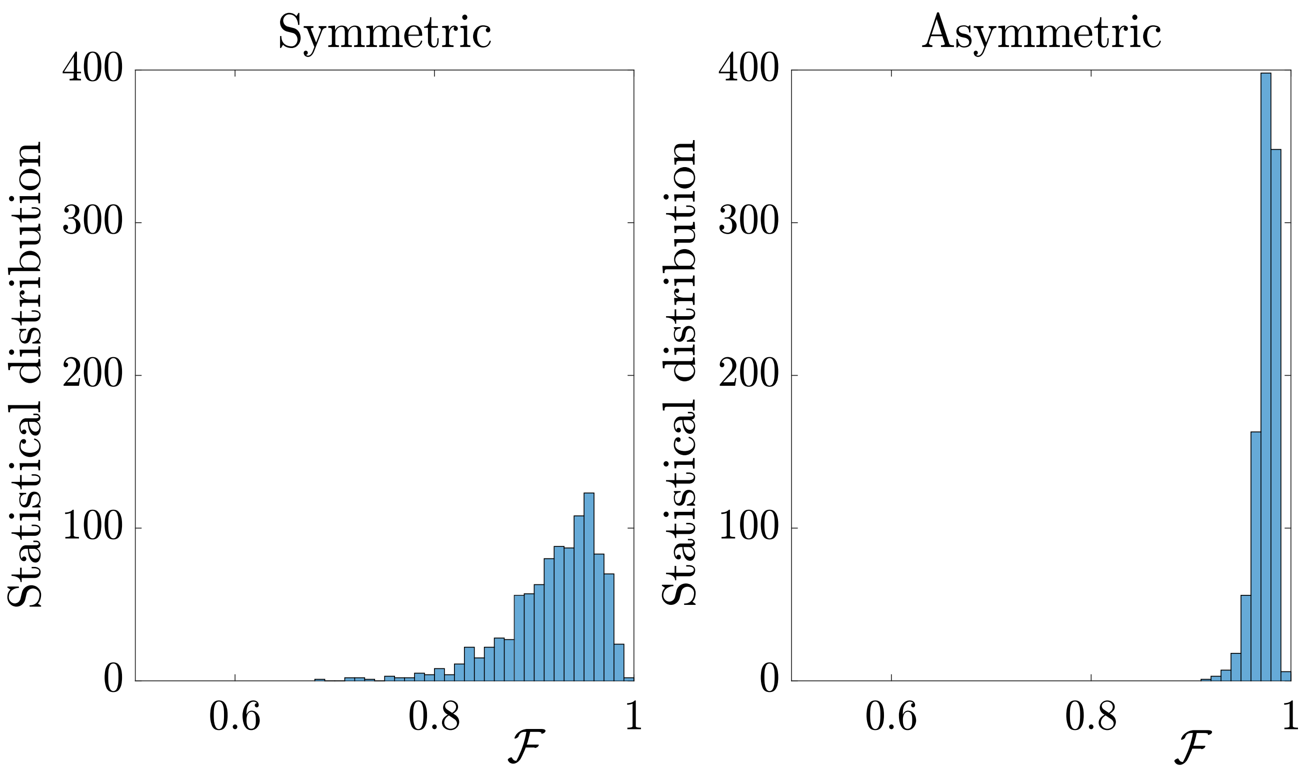




And with the optimal you get some bonus!

- Amplification: only mechanical!
- Better robustness against disorder!





What’s next…

- Understand deeper (eg. Adiabaticity) and publish!
- Link with Compact Localized States (check **Christian**’s talk)
- Link with state transfer/gates (check **Nicolas**’+ **Malte**’s talk)
- Link with non-linear stuff (check **Giorgos**’ talk)
- Link with 2D SSH stuff (check **Vassos**’s talk)
- And last but not least Do IT EXPERIMENTALLY! (check **Olivier**’s talk)